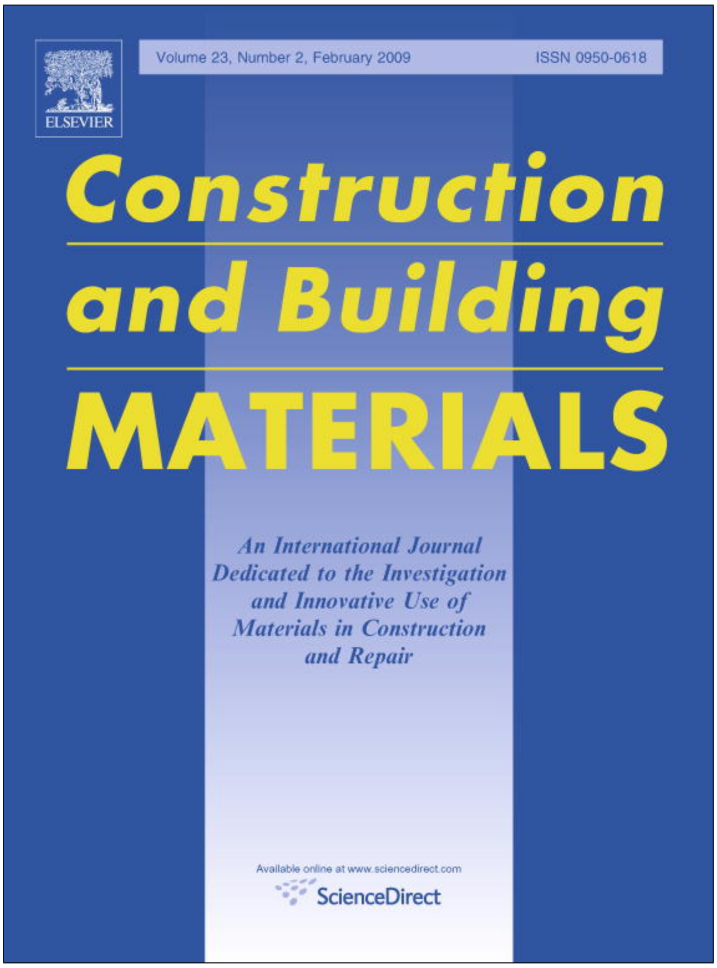

Volume 23, Number 2, February 2009
ISSN 0950-0618
ELSEVIER
Construction and Building MATERIALS
An International Journal Dedicated to the Investigation and Innovative Use of Materials in Construction and Repair
Available online at www.sciencedirect.com
ScienceDirect

Construction and Building Materials 23 (2009) 1109–1117

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

A simplified mechanical model to assess the bearing capacity of masonry walls: Theory and experimental validation

A. Mebarki [a], Q.H. Bui [a], R. Ami Saada [a,*], P. Delmotte [b], S. Sanchez Tizapa [a]

[a] *Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME FRE 3160 CNRS, 5 Bd Descartes, 77454 Marne-La-Vallée, France*
[b] *Centre Scientifique et Technique du Bâtiment (CSTB), 77447 Marne -La- Vallée cedex 2, France*

ARTICLE INFO

Article history:
Received 19 May 2006
Received in revised form 5 June 2008
Accepted 16 June 2008
Available online 8 August 2008

Keywords:
Masonry
Bearing capacity
Lateral loads
Model
Clay blocks
Concrete blocks
Joint mortar
Compressive strength
Tensile strength

ABSTRACT

This paper deals with the bearing capacity of masonry walls under lateral loads. Four different series of experimental measures have been collected, representing a total number of 20 walls tested at the Scientific and Technical Center for Buildings (CSTB, France). The constitutive materials of the walls and the geometrical features of the walls are:

- Orthotropic blocks (masonry or concrete units), with either horizontal or vertical cells. Their geometrical dimensions are such that the thickness is either equal to 0.2 m or 0.38 m while the ratios (height/length) range from 0.4 up to 1. The compressive strength of the blocks are in relative ratios (horizontal/vertical strengths) ranging from 0.11 up to 3.11.
- Joints made of mortar or thin layer mortar. The vertical joints might be either empty or full while the horizontal joints are full for the whole experiments reported herein.
- Walls with lengths ranging from 1 m up to 3.75 m while the height range from 2.5 m up to 2.8 m.

An existing model, relying on the principle of wall failure by its diagonal in compression, has herein been applied and its results have been compared with the experimental values for the 20 available walls. The model for compressive diagonal provides results that range within the interval (0.52 up to 2.67) times the experimental bearing capacity of the masonry walls.

The authors have therefore developed a simplified model that assumes that the wall fail by induced tension in the perpendicular direction of the diagonal of either the blocks or the walls. Compared to the experimental values collected in this paper, this simplified mechanical model provides theoretical bearing capacity values that are in good accordance with the observed values.

1. Introduction

Several simplified models have been investigated in order to predict the bearing capacity of masonry walls under lateral loads. Some of these models rely on the assumption that the wall failure is due to the excessive compression in the equivalent diagonal. This kind of models has therefore to address three main aspects: angle of this diagonal (among the wall diagonal or the block diagonal, mainly), the width of this equivalent diagonal and the compressive strength of this diagonal since the walls may show orthotropic behavior, [1–11]. A set of experiments, performed at CSTB (France) have led to the evaluation of these three aspects, [1].

Further experiments have been performed recently with various kinds of walls, blocks and joints, [12–15]. The validity, of the existing model, for the whole available set of walls, blocks and joints is analyzed herein.

In order to avoid the empirical evaluation of the set of parameters required by the compressive diagonal model, a new simplified mechanical model has been adopted and proposed in the present paper. It assumes that the wall failure is due to the induced tension, as a consequence of the materials heterogeneity.

2. Experimental data available for walls

2.1. Masonry walls under lateral loads and failure

Fig. 1 shows the apparatus available at CSTB and the main cracks patterns. The main features of the walls, the blocks and the joints are given in (Tables 1–4).

On Fig. 1c, one may notice that the cracks follow mainly a straight line that corresponds to either the block diagonal, in the case of empty vertical joints, or the wall diagonal, when the vertical joints are full.

---

* Corresponding author. Tel.: +33 1 60 95 77 80; fax: +33 1 60 95 77 99.
E-mail address: amisaada@univ-mlv.fr (R. Ami Saada).

0950-0618/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.conbuildmat.2008.06.004

Nomenclature

L_b	length of the block
H_b	height of the block
e_b	thickness of the block
σ_v	vertical strength of the block
σ_h	horizontal strength of the block
$\sigma_c\ (\gamma)$	diagonal compressive strength
$\sigma_t\ (\gamma)$	diagonal tensile strength
υ	tensile vs. compressive diagonal strength ratio
$\bar{\upsilon}$	mean value of the variable υ
$\bar{\bar{\upsilon}}$	standard deviation value of the variable υ
L_w	length of the wall
H_w	height of the wall
F_{exp}	experimental lateral load
F_{th}	theoretical lateral load
μ	theoretical vs. experimental lateral load ratio
$\tilde{l}_b$	width of the compressive diagonal strut
A_d	diagonal area that resist the induced tension
l_c	fitting parameter for the diagonal width
r	fitting parameter for the diagonal width
$\gamma = \gamma_d$	angle between the compressive diagonal and the vertical direction
γ_b	angle of the block diagonal
γ_w	angle of the wall diagonal
γ_0	internal friction angle of the constitutive materials (blocks + joints)

Fig. 2 shows some kinds of the hollow blocks that have been used as the constitutive materials for the tested walls. They have orthotropic properties. They may have either horizontal or vertical cells and are either clay blocks or concrete blocks.

2.2. Analysis of the collected data

From the Tables 1–4, it can be drawn, from the observed walls capacity bearing, that:

- For a given set of quality blocks and type of joints, the walls resistance decreases when the ratio (H_w/L_w) increases (Fig. 3).
- The failure pattern corresponds to cracks that cross the blocks and propagate also along some joints.

3. Model assuming wall failure by diagonal in compression

3.1. Definition of the compressive diagonal model

In order to predict the bearing capacity of masonry walls under the effect of lateral walls, a simplified model has already been issued. It relies on the hypothesis that the wall failure is due to excessive compression along a diagonal that may follow either the wall or blocks diagonals, [4,16–20] (Fig. 4). It has been developed and fitted according to the Serie-3 experiments reported in (Table 2).

The theoretical wall bearing capacity is expressed as follows, [4,19–21]:

$$F_{th} = e_b \cdot \tilde{l}_d \cdot \sigma_c(\gamma) \cdot \sin(\gamma) \tag{1}$$

$$\tilde{l}_b = l_b \cdot \left(\frac{l_b}{l_c}\right)^r \tag{2}$$

$$\sigma_c(\gamma) = \left(\frac{(\sigma_h \cdot \sigma_v)^2}{(\sigma_h \cdot \cos\gamma)^2 + (\sigma_v \cdot \sin\gamma)^2}\right)^{\frac{1}{2}} \tag{3}$$

$$\gamma = \gamma_d = \begin{cases} \mathrm{Min}\begin{cases}\gamma_b \\ \gamma_w \\ \gamma_0\end{cases} : \text{Empty joints} \\ \mathrm{Min}\begin{cases}\gamma_w \\ \gamma_0\end{cases} : \text{Full joints}\end{cases} \tag{4}$$

$$\gamma_b = \frac{H_b}{\left(\frac{L_b}{k}\right)}; \gamma_w = \frac{H_w}{L_w} \text{ and } \gamma_0 = 60^\circ \tag{5}$$

$$k = \begin{cases} 2 : \text{if Blocks Casting} = 1/2 - 1/2 \\ 3 : \text{if Blocks Casting} = 1/3 - 2/3 \end{cases} \tag{6}$$

where, F_{th} = lateral wall capacity, e_b = block thickness, $\gamma = \gamma_d$ = angle between the compressive diagonal and the vertical direction, $\sigma_c(\gamma)$ = compressive strength of the orthotropic bricks along the diagonal, σ_h = horizontal compressive strength of the bricks, σ_v = vertical compressive strength of the bricks, $\tilde{l}_b$ = width of the compressive diagonal that resist against the lateral load, l_b = block length, l_c and r = fitting parameters, γ_b = angle of the block diagonal for empty joints which depends on type of blocks casting (1/2 or 1/3 block length), H_b = block height, L_b = block length, γ_w = angle of the wall diagonal, H_w = wall height, L_w = wall length and γ_0 = internal friction angle of the constitutive materials (blocks + joints).

3.2. Analysis of the compressive diagonal model validity

The compressive diagonal model has been developed and issued by fitting the parameters to the results obtained for the set of 6 walls (Serie 3) (Table 2).

The authors have analyzed its accuracy in the case of the whole available data in order to analyze its validity:

- For other kinds of masonry blocks.
- For types of joints other than mortar.
- For various (H_W/L_W) values: “long” or “short” walls.

The theoretical bearing capacity values have been compared to the observed walls strengths, as summarized in (Tables 5 up to 8).

It can be drawn from the comparisons given in (Tables 5 up to 8), that the diagonal compressive model:

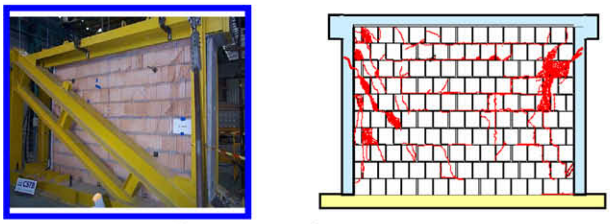
- Provides theoretical results that are in accordance with the experimental results in the case of long wall ($H_w/L_w < 1$) built with mortar joints (Tables 6 and 8). Actually, the relative error remains smaller than 20%. However, for one wall with empty vertical joints, this error equals 40% (Table 7).
- Should be improved in order to address the case of walls with large values of the ratio (H_w/L_w) and the case of thin layer mortar joints.

4. Proposal of simplified mechanical model: Failure by induced tension

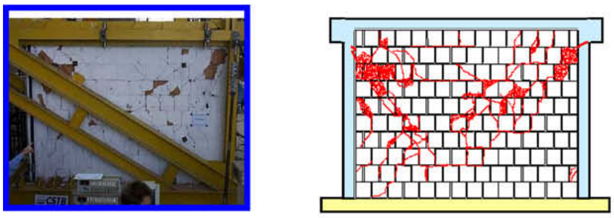
4.1. Mechanical aspects of the wall failures

According to the cracking mode on the diagonal compression test, the simultaneous compression and tension along the

(a)- Experimental devices and apparatus

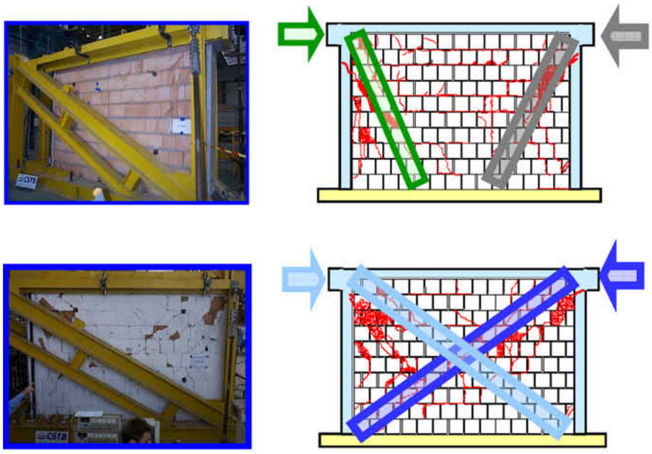


Empty vertical joint



Full vertical joint

(b)- Patterns of cracks and wall failure



(c)- Angles of the main crack lines (diagonals)

Fig. 1. Experiments on walls at CSTB.

diagonals of the wall produce a pure shear stress state which may cause the failure by cracking along the compressed diagonal, [18]. The authors consider that the cracks may be due to induced tension along the diagonal, perpendicularly to the compression direction (Fig. 5). Actually, this may due to the heterogeneities of the constitutive materials. Their Poisson coefficients are different, so that induced tension may appear.

Table 1.a
Features of Serie-1

Parameter	Unit	Wall number			
		1	2	3	4
Block					
Length: L_b	m	0.5			
Height: H_b	m	0.21			
Ratio: H_b/L_b	–	0.42			
Thickness: e_b	m	0.2			
Strength vertical: σ_v	MPa	5.8			
Horizontal: σ_h	MPa	8.39			
Ratio: σ_h/σ_v	–	1.45			
Materials	–	Clay blocks			
Blocks casting	–	1/2–1/2			1/3–2/3
Type of joint	–	Thin layer mortar			
Vertical joints	–	Empty	Full	Empty	Empty
Wall					
Length: L_w	m	1.06	1.06	1.55	3.36
Height: H_w	m	2.72			
H_w/L_w	–	2.56	2.56	1.75	0.81
Capacity: F_{exp}	kN	105	128	174	226

Table 1.b
Features of Serie-2

Parameter	Unit	Wall number 1
Block		
Length: L_b	m	0.5
Height: H_b	m	0.3
Ratio: H_b/L_b	–	0.6
Thickness: e_b	m	0.2
Strength vertical: σ_v	MPa	11.1
Horizontal: σ_h	MPa	0.92
Ratio: σ_h/σ_v	–	0.08
Materials	–	Clay blocks
Blocks casting	–	1/2–1/2
Type of joint	–	Thin layer mortar
Vertical joints	–	Empty
Wall		
Length: L_w	m	1.06
Height: H_w	m	2.79
H_w/L_w	–	2.63
Capacity: F_{exp}	kN	59

Table 2
Features of Serie-3

Parameter	Unit	Wall number					
		1	2	3	4	5	6
Block							
Length: L_b	m	0.5	0.5	0.25	0.25	0.5	0.5
Height: H_b	m	0.2	0.2	0.25	0.25	0.25	0.25
Ratio: H_b/L_b	–	0.4	0.4	1	1	0.5	0.5
Thickness: e_b	m	0.2	0.2	0.38	0.38	0.2	0.2
Reference	–	T1	T1	T2	T2	T3	T3
Strength vertical: σ_v	MPa	2.7	2.7	19.6	19.6	12.8	12.8
Horizontal: σ_h	MPa	8.39	8.39	5.43	5.43	1.4	1.4
Ratio: σ_h/σ_v	–	3.11	3.11	0.28	0.28	0.11	0.11
Materials	–	Clay blocks					
Blocks casting	–	1/2–1/2					
Type of joint	–	Mortar					
Vertical joints	–	Full	Empty	Full	Empty	Full	Empty
Wall							
Length: L_w	m	3.45					
Height: H_w	m	2.63	2.63	2.61	2.61	2.61	2.61
H_w/L_w	–	0.76	0.76	0.76	0.76	0.76	0.76
Capacity: F_{exp}	kN	432	374	504	306	210	167

Table 3
Features of Serie-4

Parameter	Unit	Wall number			
		1	2	3	4
Block					
Length: L_b	M	0.63			
Height: H_b	M	0.25			
Ratio: H_b/L_b	–	0.4			
Thickness: e_b	M	0.2			
Strength vertical: σ_v	Mpa	4	4	5.38	5.38
Horizontal: σ_h	Mpa	2	2	2.69	2.69
Ratio: σ_h/σ_v	–	0.5	0.5	0.5	0.5
Blocks casting	–	1/2-1/2			
Type of joint	–	Thin layer mortar			
Vertical joints	–	Full	Empty	Full	Empty
Wall					
Length: L_w	M	3.71	3.73	3.71	3.72
Height: H_w	M	2.5			
H_w/L_w	–	0.67			
Capacity: F_{exp}	KN	227	204	233	201

Table 4
Features of Serie-4 (concrete blocks series)

Parameter	Unit	Wall number				
		1	2	3	4	5
Block						
Length: L_b	m	0.50				
Height: H_b	m	0.20				
Ratio: H_b/L_b	–	0.40				
Thickness: e_b	m	0.20				
Reference	–	Concrete block				
Strength vertical: σ_v	MPa	9.30				
Horizontal: σ_h	MPa	4.65				
Ratio: σ_h/σ_v	–	0.50				
Blocks casting	–	1/2–1/2				
Type of joint	–	Mortar	Mortar	Mortar	Mortar	Thin layer mortar
Vertical joints	–	Full	Empty	Full	Empty	Empty
Wall						
Length: L_w	m	3.74	3.75	3.71	3.70	3.71
Height: H_w	m	2.61	2.61	2.61	2.61	2.65
H_w/L_w	–	0.70	0.70	0.70	0.71	0.71
Capacity: F_{exp}	kN	556	429	534	380	480

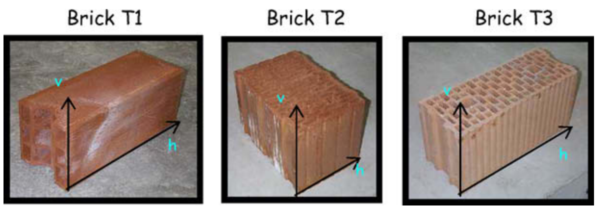
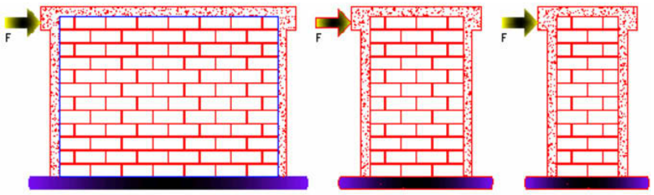


Fig. 2. Some hollow blocks used for the walls.



(a)- Small (b)- medium (c)- Large

Fig. 3. Various (H_w/L_w) ratio values.

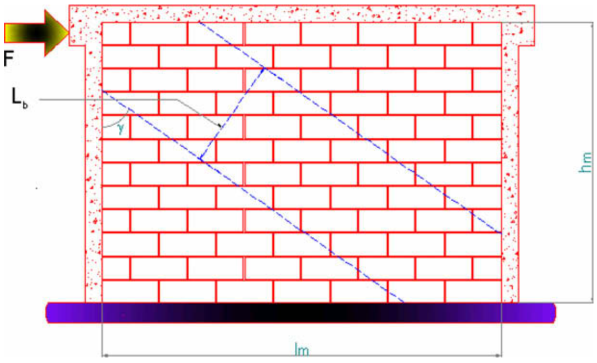


Fig. 4. Compressive diagonal model.

4.2. Proposal of a simplified mechanical model

The existing compressive diagonal model, that is widely used, has to address the main following topics: angle of the diagonal, compressive strength of the diagonal, equivalent diagonal width

Table 5.a
Theoretical vs. experimental walls bearing capacities: Serie-1

Parameter	Unit	Wall number			
		1	2	3	4
Type of joint	–	Thin layer mortar			
Vertical joints	–	Empty	Full	Empty	Empty
Wall					
Length: L_w	m	1.06	1.06	1.55	3.36
H_w/L_w	–	2.56	2.56	1.75	0.81
Capacity: F_{exp}	kN	105	128	174	226
Compressive diagonal model					
Angle: γ	[°]	21.3	21.3	29.7	38.4
$\mu = F_{th}/F_{exp}$	–	**2.63**	**2.51**	**2.23**	**2.25**

Table 5.b
Theoretical vs. experimental walls bearing capacities: Serie-2

Parameter	Unit	Wall number		
		1	2	3
Type of joint	–	Thin layer mortar		
Vertical joints	–	Empty	Empty	Full
Wall				
length: L_w	m	1.06	3.75	3.75
H_w/L_w	–	2.63	0.74	0.74
Capacity: F_{exp}	kN	59	222	175
Compressive diagonal model				
Angle: γ	[°]	20.8	29.1	53.3
$\mu = F_{th}/F_{exp}$	-	**1.92**	**0.52**	**0.77**

Table 7
Theoretical vs. experimental walls bearing capacities: Serie-4

Parameter	Unit	Wall number			
		1	2	3	4
Type of joint	–	Thin layer mortar			
Vertical joints	–	Full	Empty	Full	Empty
Wall					
Length: L_w	m	3.71	3.73	3.71	3.72
H_w/L_w	–	0.67	0.67	0.67	0.67
Capacity: F_{exp}	kN	227	204	233	201
Compressive diagonal model					
Angle: γ	[°]	56	51.3	56	51.3
$\mu = F_{th}/F_{exp}$	–	**1.73**	**1.75**	**2.27**	**2.38**

that resists the lateral load. This diagonal width requires an empirical determination.

The proposed alternative model suggests that the diagonal resistance against the induced tension should be expressed as follows:

$$F_{th} = A_d \cdot \sigma_t(\gamma) \cdot f(\gamma) \tag{7}$$

$$A_d = e_b \cdot \left(\frac{H_w}{\cos(\gamma)}\right) \tag{8}$$

$$\sigma_t(\gamma) = \upsilon \cdot \sigma_c(\gamma) \tag{9}$$

$$f(\gamma) = \cot g(\gamma) \tag{10}$$

$$\sigma_c(\gamma) = \begin{cases} \left(\frac{(\sigma_h \cdot \sigma_v)^2}{(\sigma_h \cdot \cos\gamma)^2 + (\sigma_v \cdot \sin\gamma)^2}\right)^{\frac{1}{2}} : \text{Elliptic} \\ \text{or} \\ \left(\frac{\sigma_h \cdot \sigma_v}{\sigma_h \cdot \cos\gamma + \sigma_v \cdot \sin\gamma}\right) : \text{Linear} \end{cases} \tag{11}$$

where, F_{th} = lateral wall capacity, e_b = block thickness, $\gamma = \gamma_d$ = angle between the compressive diagonal and the vertical direction, $\sigma_c(\gamma)$ = compressive strength of the orthotropic bricks along the diagonal, σ_h = horizontal compressive strength of the bricks, σ_v = vertical compressive strength of the bricks, A_d = diagonal area that resist the induced tension, $f(\gamma)$ is a trigonometric function that may be considered in order to express the resistance in tension of the diagonal under an the horizontal load, $f(\gamma) = \cot g(\gamma)$ is adopted as the best fitting function among other trigonometric functions (sin, 1/sin, cos, 1/cos, tg) and v is the ratio between the strength in tension σ_t and the compressive strength σ_c. The authors have assumed herein that the diagonal compressive strength, σ_c, may be derived from the two orthotropic strength, Eq. (10), by either an elliptic relationship (as considered in Eq. (3)) or a linear relationship.

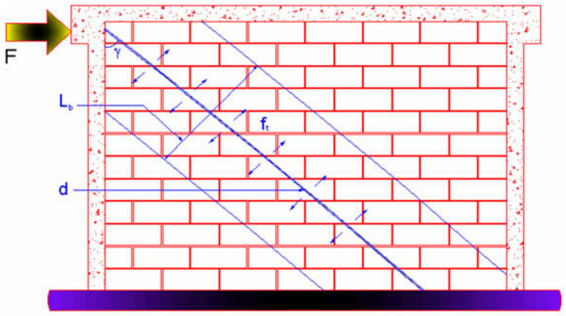
The present mechanical model addresses the case of wall failure by diagonal cracks, omitting in the present step the case of failure by shear.

Table 6
Theoretical vs. experimental walls bearing capacities: Serie-3

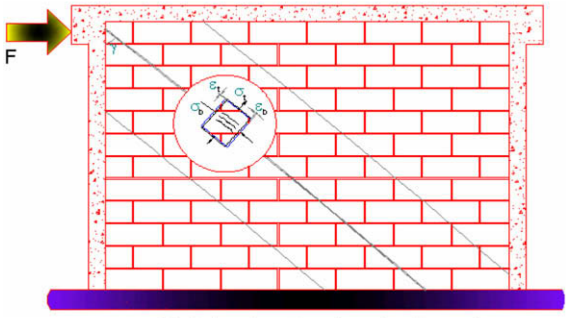
Parameter	Unit	Wall number					
		1	2	3	4	5	6
Type of joint	–	Mortar					
Vertical joints	–	Full	Empty	Full	Empty	Full	Empty
Wall							
Length: L_w	m	3.45	3.45	3.45	3.45	3.45	3.45
H_w/L_w	–	0.76	0.76	0.76	0.76	0.76	0.76
Capacity: F_{exp}	kN	432	374	504	306	210	167
Compressive diagonal model							
Angle: γ	[°]	52.7	51.3	52.9	26.6	52.9	45
$\mu = F_{th}/F_{exp}$	–	**1.11**	**1.06**	**1.00**	**1.00**	**0.98**	**1.05**

Table 8
Theoretical vs. experimental walls bearing capacities: Serie-5

Parameter	Unit	Wall number				
		1	2	3	4	5
Type of joint	–	Mortar	Mortar	Mortar	Mortar	Thin layer mortar
Vertical joints	–	Full	Empty	Full	Empty	Empty
Wall						
Length: L_w	m	3.74	3.75	3.71	3.70	3.71
H_w/L_w	–	0.70	0.70	0.70	0.71	0.71
Capacity: F_{exp}	kN	556	429	534	380	480
Compressive diagonal model						
Angle: γ	[°]	55.1	51.3	54.9	51.3	51.3
$\boldsymbol{\mu = F_{th}/F_{exp}}$	–	**1.16**	**1.27**	**1.21**	**1.43**	**1.13**



(a)- Wall cracks along the diagonal



(b)- Induced tension along the diagonal

Fig. 5. Induced tension along the diagonal.

Table 9a
Induced tension model vs. experimental walls bearing capacities: Serie-1

Parameter	Unit	Wall number			
		1	2	3	4
Reference	–	*04147*	*04248*	*05002*	*04133*
Type of joint	–	Thin layer mortar			
Vertical joints	–	Empty	Full	Empty	Empty
Wall					
Length: L_w	m	1.06	1.06	1.55	3.36
H_w/L_w	–	2.56	2.56	1.75	0.81
Capacity: F_{exp}	kN	105	128	174	226
Induced tension model					
Diagonal:					
Angle: γ	[°]	21.3	21.3	29.7	38.4
Strength: $\sigma_c(\gamma)$	MPa	4.9	4.9	4.79	4.78
Length: L_{diag}	m	2.92	2.92	3.13	3.47
Area: S_{diag}	m^2	0.58	0.58	0.63	0.69
$\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$	–	**0.09**	**0.11**	**0.10**	**0.09**

Table 9b
Induced tension model vs. experimental walls bearing capacities: Serie-2

Parameter	Unit	Wall number 1
Type of joint	–	Thin layer mortar
Vertical joints	–	Empty
Wall		
Length: L_w	m	1.06
H_w/L_w	–	2.63
Capacity: F_{exp}	kN	59
Induced tension model		
Diagonal		
Angle: γ	[°]	20.8
Strength: $\sigma_c(\gamma)$	MPa	2.53
Length: L_{diag}	m	2.99
Area: S_{diag}	m^2	0.60
$\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$	–	**0.12**

4.3. Analysis of the induced tension model validity

The authors have analyzed its accuracy in the case of the whole available data. The theoretical bearing capacity values have been compared to the observed walls strengths, as summarized in (Tables 9–12). They correspond to the linear form of Eq. (11). One should notice that for the whole walls under study, the case of empty vertical joints leads to smaller bearing capacity when compared to the case of full vertical joint. In the case of linear relationship between the diagonal and the orthotropic compressive strengths, the remaining results show that the ratio $\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$ ranges within the interval:

- [0.09–0.12] for Serie-1 and Serie-2.
- [0.04–0.13] for Serie-3. In fact, three kinds of masonry blocks are considered for these sets. For each set of blocks, the ratio remains almost the same except for the type T2 of the blocks.
- [0.07–0.11] for the Serie-4.
- [0.10–0.11] for Serie-5.

The values obtained for the ratio $\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$ are summarized in (Tables 9a, 9b, 10–12) with a mean value:

$$\overline{v} = 0.1,\ \text{i.e.}\ \sigma_t(\gamma) = \overline{v}\cdot\sigma_c(\gamma) \tag{12}$$

The theoretical diagonal resistance against the induced tension, derived from Eqs. (7)–(9) becomes therefore:

$$F_{th} = A_d\cdot\overline{v}\cdot\sigma_c(\gamma)\cdot f(\gamma) \tag{13}$$

The ratio (μ) between the theoretical and experimental bearing capacities of the masonry wall is therefore defined by

$$\mu = F_{th}/F_{exp} \tag{14}$$

Fig. 6. shows that the induced tension model has a good efficiency as more than 90% of this ratio values range within the interval [0.75–1.25], whereas the compressive diagonal model provides only 45% efficiency within the same interval (Fig. 6). However,

Table 10
Induced tension model vs. experimental walls bearing capacities: Serie-3

Parameter	Unit	Wall number					
		1	2	3	4	5	6
Type of joint	–	Mortar					
Vertical joints	–	Full	Empty	Full	Empty	Full	Empty
Wall							
Length: L_w	m	3.45	3.45	3.45	3.45	3.45	3.45
H_w/L_w	–	0.76	0.76	0.76	0.76	0.76	0.76
Capacity: F_{exp}	kN	432	374	504	306	210	167
Induced tension model							
Diagonal							
Angle: γ	[°]	52.7	51.3	52.9	26.6	52.9	45
Strength: $\sigma_c(\gamma)$	MPa	3.13	3.08	5.63	7.82	1.62	1.78
Length: L_{diag}	m	4.34	4.21	4.33	2.92	4.33	3.69
Area: S_{diag}	m^2	0.87	0.84	1.62	1.09	0.87	0.74
$\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$	–	**0.12**	**0.12**	**0.04**	**0.07**	**0.11**	**0.13**

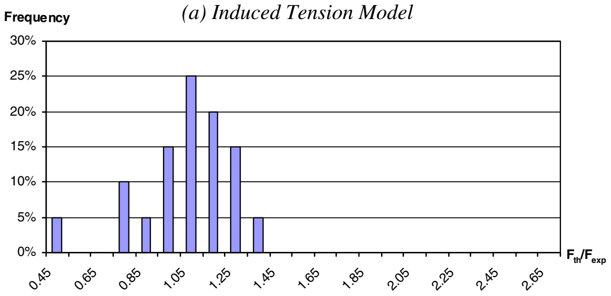
Table 11
Induced tension model vs. experimental walls bearing capacities: Serie-4

Parameter	Unit	Wall number			
		1	2	3	4
Type of joint	–	Thin layer mortar			
Vertical joints	–	Full	Empty	Full	Empty
Wall					
Length: L_w	m	3.71	3.73	3.71	3.72
H_w/L_w	–	0.67	0.67	0.67	0.67
Capacity: F_{exp}	kN	227	204	233	201
Induced tension model					
Diagonal					
Angle: γ	[°]	56	51.3	56	51.3
Strength: $\sigma_c(\gamma)$	MPa	1.8	1.83	2.43	2.46
Length: L_{diag}	m	4.47	4.00	4.47	4.00
Area: S_{diag}	m^2	0.9	0.8	0.9	0.8
$\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$	–	**0.09**	**0.11**	**0.07**	**0.08**

more sophisticated mechanical models are still required for other typologies of masonry walls: other kinds of blocks, various joints quality, presence of openings, presence of reinforced concrete or metal frames, etc.

An error model is therefore considered for the following reasons:

- Heterogeneity and uncertainty in the materials properties.
- Simplified approach of the walls mechanical behaviour, i.e. the failure by induced tension in the diagonal.


(a) Induced Tension Model

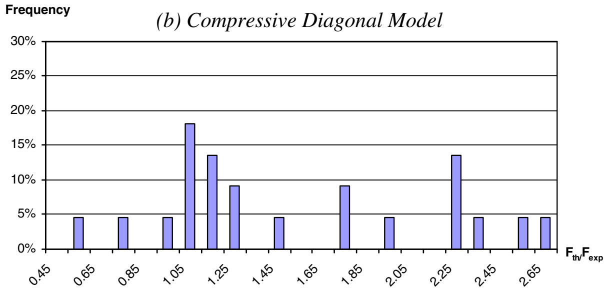

(b) Compressive Diagonal Model

Fig. 6. Comparison between theoretical and experimental results ((a) induced tension model and (b) compressive diagonal model).

- Simplified relation between the tensile and compressive strengths.

Many error models are widely used, [22–25]. Further data are required in order to establish the most adequate error model. However, we assume, at the present stage, that the error model may be adequately described by one among the following distributions:

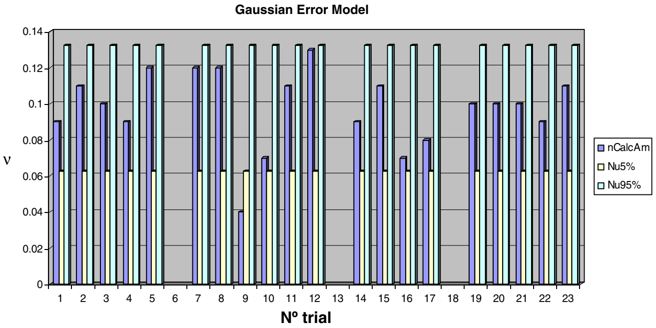
- The gamma distribution.
- The log-normal distribution.
- The normal distribution.

According to the results, reported in (Tables 9–12), the distribution of the ratio v is so that

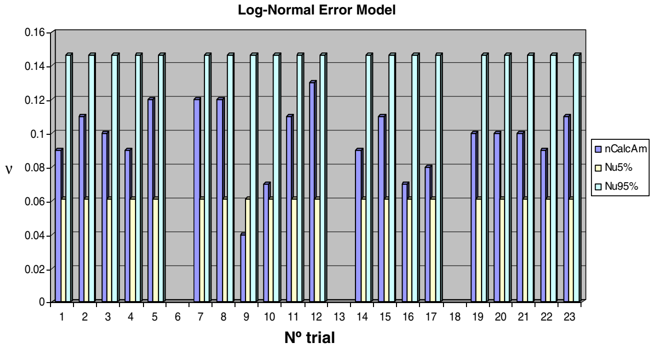
- The mean value $\overline{v} = 0.1$.
- The standard deviation value $\overline{\overline{v}} = 0.02$.
- The coefficient of variation (c.o.v.) $C_v = 20\%$. One should notice that a close value of 15% of c.o.v. is commonly admitted for the materials properties such as compressive strengths, [22].

Table 12
Induced tension model vs. experimental walls bearing capacities: Serie-5

Parameter	Unit	Wall number				
		1	2	3	4	5
Type of joint	–	Mortar	Mortar	Mortar	Mortar	Thin layer mortar
Vertical joints	–	Full	Empty	Full	Empty	Empty
Wall						
Length: L_w	m	3.74	3.75	3.71	3.70	3.71
H_w/L_w	–	0.70	0.70	0.70	0.71	0.71
Capacity: F_{exp}	kN	556	429	534	380	480
Induced tension model						
Diagonal						
Angle: γ	[°]	55.1	51.3	54.9	51.3	51.3
Strength: $\sigma_c(\gamma)$	MPa	4.20	4.25	4.21	4.25	4.25
Length: l_{diag}	m	4.56	4.18	4.54	4.18	4.24
Area: S_{diag}	m^2	0.91	0.84	0.91	0.84	0.85
$\boldsymbol{v = \sigma_t(\gamma)/\sigma_c(\gamma)}$	–	**0.10**	**0.10**	**0.10**	**0.10**	**0.11**



(a)- Gaussian Error Model



(b)- Log-Normal Error Model

Fig. 7. Ratios v and the quantiles for a normal distribution of the error model: Linear form of $\sigma_t\,(\gamma)$.

We adopt therefore for the ratio v:

- Two kinds of distribution that are widely used once the mean and the standard deviation values are known, i.e. a gaussian and a log-normal, with a mean value $\overline{\upsilon} = 0.1$ and a c.o.v. $C_V = 20\%$.
- The fractiles 5% and 95% of the distribution expressed as:

$$\upsilon_{5\%} = \mu_\upsilon \cdot (1 - 1.645 C_\upsilon) \quad (15)$$

$$\upsilon_{95\%} = \mu_\upsilon \cdot (1 + 1.645 C_\upsilon) \quad (16)$$

The theoretical values of the ratio v, given in (Tables 9a, 9b, 10–12), are compared to the intervals $[v_{5\%}:v_{95\%}]$ (Fig. 7).

Except for the blocks T2 with empty vertical joints in Serie-3, the experimental values of the ratio v are in accordance with the theoretical interval $[v_{5\%}:v_{95\%}]$, for the both distributions that are adopted for the error model.

5. Conclusion

This article is based on the experimental data collected for 20 walls tested at CSTB France: many walls slenderness, many kinds of hollow blocks (masonry, concrete) and various types of joints (mortar and Thin layer mortar, empty or full vertical joints).

The walls have been analyzed under the effect of a lateral load. The patterns of cracks have been studied and the orthotropic compressive strengths of the blocks have been measured.

The compressive diagonal model has been considered. Its results show a great difference with the walls bearing capacities, in many cases. This model requires, in fact, a set of fitting parameters that need to be adapted for each set of walls, bricks and joints.

A new model, called the induced tension model, has been proposed herein. It assumes that the materials heterogeneity's give rise to induced tension along the diagonal. When the tensile stress generated by induced tension along the diagonal overcomes the tensile strength of this diagonal, cracks appear causing the wall failure.

This model relies on the following parameters:

- Direction of the diagonal: the angle depends on the walls slenderness (H_w/L_w), blocks slenderness (H_b/L_b) and the kind of vertical joints, as well as the internal friction angle.
- Compressive strength of the diagonal: elliptic and linear relationships are assumed in order to obtain this strengths from the orthotropic compressive strengths of the blocks.
- Tensile strength of the diagonal: a simplified relationship has been assumed between this strength and the compressive diagonal strength.
- Error model that takes also into account the materials heterogeneities.

Under these hypotheses, the collected results show that:

- The induced tension model provides walls bearing capacities that are in good accordance with the observed values for a wide range of walls dimensions, blocks dimensions, kinds of joints (empty or full), constitutive materials for both of the blocks (concrete or clay units) and the joints (mortar or thin layer) and blocks anisotropy (vertical or horizontal cells for the hollow blocks). More than 90% of the ratio between the theoretical and experimental bearing capacities range within the interval [0.75–1.25].
- The linear relationship between the diagonal strength and the orthotropic blocks strengths provides good results.
- Both Gaussian and log-normal error model distribution with a 20% of c.o.v. on the materials quality provide theoretical intervals (5% and 95% quantiles) in which fall the theoretical predictions of the mechanical model, except for one kind of blocks when the vertical joint is empty.

This model might be therefore improved in order to consider the presence of a vertical load on the wall and also predict the ultimate strength according to two main causes of failure: induced tension on the diagonal or shear on horizontal planes.

References

[1] Cruz Diaz JI, Etude des murs de contreventement en maçonnerie d'éléments de terre cuite, Génie Civil. Thesis. France: Université de Marne La Vallé; 2000.
[2] Cruz Diaz JI, Sellier A, Capra B, Delmotte P, Rivillon P, Mébarki A. Modélisation simplifiée du comportement à rupture des murs. RFGC – Revue Française de Génie Civil, 2001;5:613–27 [Ed Hermès].
[3] Capra B, Cruz-Diaz JI, Delmotte P, Mébarki A, Rivillon P, Sellier A. Murs de contreventement en maçonnerie de terre cuite: approche expérimentale et modélisation du comportement à rupture. Cahiers Techniques du CSTB Etudes et Recherches – Cahier No. 3310, Paris; 2001.
[4] Cruz Diaz JI, Sellier A, Capra B, Delmotte P, Rivillon P, Mébarki A. Resistance of masonry wind braced walls: simplified model and experimental validation. Masonry Int J 2002;15(3):73–9 [UK: British Masonry Society].
[5] Abdou L. Modélisation du comportement mécanique des murs en maçonnerie chargés dans leur plan, Génie Civil. Thesis. France: Université de Marne La Vallé; 2005.
[6] Lafuente M. Contribution à l'étude analytique du comportement de murs en maçonnerie non armée sous sollicitation plane, Génie Civil. Thesis. France: INSA Toulouse; 1991.
[7] Abdou L, Ami Saada R, Meftah F, Mébarki A. On the sliding behaviour of the brick-mortar interface: an experimental study. Masonry Int J 2004;17(3):129–34 [UK: British Masonry Society].
[8] Abdou L, Ami Saada R, Meftah F, Mébarki A. Experimental investigation of the joint mortar behaviour. Mech Res Commun 2006;33:370–84.
[9] Abdou L, Meftah F, Ami Saada R, Mébarki A. Effet des conditions aux limites sur le mode de rupture des murs en maçonnerie. In: Proceedings of Colloque International "Risque Vulnérabilité et fiabilité dans la construction. Vers une réduction des désastres" Alger; 2003. p. 280-289. ISBN 9961-891-01-5.

[10] Abdou L, Ami Saada R, Meftah F, Mébarki A. Cisaillement de la maçonnerie: Aspects expérimentaux et modélisation, Congrès Français de Mécanique (*CFM'05*), Troyes, France; 2005. 5p.

[11] Corradi M, Borri A, Vignoli A. Experimental study on the determination of strength of masonry walls. Constr Build Mater 2003;17(5):325–37.

[12] Rapport d'étude No. ES 553 03 0142 (CSTB); France.

[13] Rapport d'étude No. ES 553 04 0181 (CSTB); France.

[14] Delmotte P, Rivillon P, Wesierski V, Hurez M. Etude des murs de contreventement en maçonnerie de blocs creux en béton. Cahiers Techniques du CSTB Etudes et Recherches – Cahier No. 3491, Paris; 2003.

[15] Delmotte P, Rivillon P, Wesierski V, Hurez M. Etude des murs de contreventement en maçonnerie de blocs en béton cellulaire auto clavé. Cahiers Techniques du CSTB Etudes et Recherches – Cahier No. 3491, Paris; 2003.

[16] El-Dakhakhni WW, Hamid AA, Hakam ZHR, Elgaaly M. Hazard mitigation and strengthening of unreinforced masonry walls using composites. Compos Struct 2006;73(4):458–77.

[17] Gabor A, Bennani A, Jacquelin E, Lebon F. Modelling approaches of the in-plane shear behaviour of unreinforced and FRP strengthened masonry panels. Compos Struct 2006;74(3):277–88.

[18] Gabor A, Ferrier E, Jacquelin E, Hamelin P. Analysis and modelling of the in-plane shear behaviour of hollow brick masonry panels 2006;20(5):308–21.

[19] Cruz Diaz JI, Sellier A, Capra B, Delmotte P, Rivillon P, Mébarki A. Fiabilité des murs de contreventement en maçonnerie: calibration des coefficients partiels d'un modèle simplifié. RFGC – Revue Française de Génie Civil 2002:7 [Ed Hermès].

[20] Cruz Diaz JI, Sellier A, Capra B, Delmotte P, Rivillon P, Mébarki A. Modélisation simplifiée du comportement à rupture des murs. RFGC – Revue Française de Génie Civil 2001;5:613-627 [Ed Hermès].

[21] Garcès F, Mébarki A, Genatios C, Lafuente M. Identification des rigidités résiduelles de systèmes à murs porteurs chaînés. Revue Française de Génie Civil 2004;8(8):889–903 [Ed Hermès].

[22] Mébarki A. Modèle d'atténuation sismique: prédiction probabiliste des pics d'accélération. RFGC – Revue Française de Génie Civil 2004;8(9–10):1071–1086 [Ed Hermès].

[23] Mébarki A, Valencia N. Informal masonry structures: seismic vulnerability and GIS maps. Masonry Int J 2004;17(1):18–25 [UK: British Masonry Society].

[24] Mébarki A. A comparative study of different PGA attenuation and error models: case of 1999 Chi-Chi earthquake, Tectonophys doi:10.1016/j.tecto.2007.11.026.

[25] Mébarki A, Nguyen QB, Mercier F, Ami Saada R, Reimeringer M. Reliability analysis of metallic targets under metallic rods impact: towards a simplified probabilistic approach. J Loss Prevention Process Industries, in press. doi:10.1016/j.jlp.2008.04.002 [Available online 26 April 2008].