Motivic topologies: mathematical and computational modelling in music analysis

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Abstract. This paper discusses a mathematical model together with its computational realization, for the motivic analysis of a piece of music. Relations between the mathematical model (motivic topologies), computational counter-part (OM-Melos), and music analysis are presented in the light of general concepts of computational music analysis, stressing the importance of neutrality and scientific rigour in the modelling part, while preserving the freedom of the analyst.

Keywords: Motivic topologies, motivic analysis, mathematical modelling, computational music analysis

1 Introduction

Mathematical modelling in music addresses topics ranging from traditional music theory to more contemporary theories of gesture and performance. A mathematical model however is an abstract construct, which needs a computational counterpart in order to be applicable to music analysis. Computational music analysis (CMA) is an area of research which uses computational means for music analysis purposes. Its aims are to produce musicologically interesting results and formalize the human analytical process, while assisting the analyst with data, calculations, and making explicit analytical choices. The whole modelling process, be it mathematical and/or computational, in order to be meaningful needs to remain close to the main questions of music analysis. In this paper, we exemplify some of the challenges in this enterprise.

2 The Mathematical and Computational Models

2.1 Motivic Spaces: the Mathematical Model.

We briefly enumerate the main concepts of the model; see [1, 2] for details and examples. A topological space on the infinite set MOT of all theoretical motives is constructed: A **motif** M of cardinality n is a non-empty finite set of n notes with all different onsets. A set mapping t on MOT, called **shape type**, a group P action on motive shapes leading to **gestalts** (imitation classes), and pseudo-metrics (similarity functions) d_n on motive shapes with fixed cardinalities n, retracted to motives as $gd_t^P(M,N) := \inf_{p,q \in P} d_n(p \cdot t(M), q \cdot t(N))$, are introduced. This leads to a topological space on each set MOT_n of motives of cardinality n. But a crucial step for regrouping motives of different cardinalities is the introduction given a **similarity threshold** $\epsilon > 0$, of the sets: $V_\epsilon^{t,P,d}(M) := \{N \in MOT | N^* \subset N \text{ s.t. } gd_t^P(N^*, M) < \epsilon\}$. If the inheritance property is fulfilled [1], the collection of all $V_\epsilon^{t,d,P}(M)$ forms a basis for a topology $\mathcal{T}_{t,P,d}$ on MOT. In this space, the **ϵ-variations** sets, $Var_\epsilon^{t,d,P}(M) := \{N \in MOT | N \in V_\epsilon^{t,P,d}(M) \text{ or } M \in V_\epsilon^{t,P,d}(N)\}$, conceptualize variations of motives.

The **motivic space of a piece** S is the relativization of $\mathcal{T}_{t,P,d}$ to $MOT(S)$, an arbitrary finite collection of motives in S, and represents the motivic structure of S [1]. Motivic analysis schemes are further modeled: (1) The **identification of germinal motives**, as proposed by Rudolph Réti [3], is modeled by quantifying $Var_\epsilon^{t,d,P}(M)$ sets through a weight function [1]; (2) **Paradigmatic categorizing**, the first stage of semiotic analysis as proposed by Nattiez [4], is realized using $Var_\epsilon^{t,d,P}(M)$ sets [5]; and (3) an **indirect temporal distribution of motivic paradigms**, is proposed by extending the weight function to notes [2].

2.2 OM-Melos: The Computational Model.

Applying the mathematical model to motivic analysis means to explicitly construct a motivic space of a piece together with the analysis scheme models. The following describes the computational model implementation ('OM-Melos' [5]).[1]

1. **Data representation** The manipulation of data is symbolic (using MIDI as the input format). The piece S is reduced to its set of notes: (onset, pitch).
2. **Segmentation.** Given the segmentation preference (using large sections or a time window), the set $MOT(S)$ of motives in S for the analysis is computed.
3. **Choice of knowledge representation of motives.** Given the shape type t selected by the analyst, that is the musical parameter(s) the analysis will focus on, OM-Melos computes the shape of each motif in $MOT(S)$.
4. **Motivic grouping into gestalts.** Given the (paradigmatic) group P selected by the analyst, motives are regrouped with their imitations (gestalts).
5. **Motivic similarity.** Given a similarity function d, the distance between any two gestalts of motives in $MOT(S)$ with same cardinality is calculated.
6. Futher motivic analysis procedures
   (a) **Calculation of weights.** Given a weight function, the weight of each gestalt in $MOT(S)$ and of each note in S are calculated.
   (b) **Paradigmatic categorization procedure.** Given a collection $X \subset MOT(S)$ of motives in S, the paradigmatic categorization is calculated.
7. **Result production.** Intermediate and final results (of diverse types, e.g. numerical, graphic, and music) are returned to the analyst for interpretation.

[1] It is a complete-model, stand-alone version of MeloRUBETTE in RUBATO [6].

3 Related Issues in Computational Music Analysis

The above procedure describes the steps in the present computational approach. Below, some related key concepts of computational music analysis are discussed.

- **Segmentation:** Any set of motives in a piece could theoretically be set as $MOT(S)$; e.g. all motives in a piece. But, this would include potentially absurd note combinations, musically meaningless (e.g. the 2-note motif comprising the very first and last note of a piece). The choice of the segmentation is carried out manually and relies on the judgement of the analyst. Overlapping segmentation is allowed, as well as segments that span across voices.
- **The concept of motif:** It is a particularly general one. Theoretically, a motif can be any combination of notes (with different onsets) taken from the piece. However, in the realization of the piece, only selected motifs through the segmentation are taken into account (Step 2). A motif may also comprise non-consecutive notes in the piece.
- **Knowledge representation:** The representation of information (or knowledge) for use in intelligent problem solving lies at the core of any computational modelling approach, almost forming an independent area of study within Artificial Intelligence. In music processing, this defines not only what musical information is represented, but also how this is done. There can be several parameters extracted, at different levels of abstraction. In this case, they are: onset, pitch, duration, and loudness, which can be considered either alone, or in combinations. Further derived representations, including interval and contour functions are calculated on any of the parameters, defining thus the shape of the motif (Step 3). Knowledge representation allows motives with the same representation to be grouped together. In this case, the similarity of the musical surface (different motives) is based on the identity on the musical parameter level.
- **The concept of similarity and categorization:** This approach allows for several levels of motivic groupings and categorizations, based on various types of similarity relations and criteria: *Level 1* is realized through the different abstract knowledge representations, as explained above (Step 3). Motives with the same representation are considered identical for further analysis; *Level 2* takes place when constructing the gestalts (Step 4). Motives with the same gestalts are considered identical for further analysis; *Level 3* is related to distances between gestalts, according to a chosen similarity function and threshold (Step 5). In this level, which allows variations, only motives with the same cardinality are considered; *Level 4* is directly related to the formalization of variations of motives (as involved in Step 6a). Motives with different cardinalities are now considered and regrouped in ϵ-variations Var_ϵ, according to a similarity threshold ϵ.

4 Theoretical implications and conclusions

In this paper we discussed connections between the mathematical and the computational model of motivic topologies, as applied to music analysis. While similar concepts exist in the two disciplines, mathematics and informatics, their relation is not always straightforward. To conclude, we bring up two issues that we consider important for this type of collaborative interdisciplinary research.

a. Scientific rigour and objectivity: A model is by definition rigorous and scientific. The analysis procedure should thus be reproducable as long as the parameters used by the analyst are taken into account. One related question is whether the process of music analysis, following a model, can also be thought of as "neutral" (in Nattiez' terms) and objective[2]. When looking at the described mathematical model, it first proposes an abstract construct on an infinite set of theoretical motives. It then carries the topological structure to "an arbitrary finite set of motives". Whether the actual choice of the "arbitrary" set of motives for the analysis of a piece makes sense, or the choice of analysis parameters is appropriate, one possible answer is that as long as the points of choice for the analyst are made explicit, the analytical process is formalized, and therefore perhaps closer to what Nattiez had originally envisaged.

b. Analytical freedom: It is clear that the freedom of choice given to the analyst is a significant aspect of the model in all its steps. Since the model is of generic nature, one would wish to vary the parameters for a more complete and context-free analysis. As in many computational music analysis approaches, the proposed model involves a similarity threshold in its procedures. Should this analytical parameter also be made accessible to the analyst (although it is not clear on which criteria the analyst would base his choice)? This turns out to be a key aspect of the approach: due to the diverse visualizations of results, the analyst does not have to select a similarity threshold a priori, but instead can use the overall motivic spectrum, e.g. the paradigmatic categorization dynamic plots, and select, a posteriori, which similarity thresholds should be considered based on the meaningfulness of their corresponding results.

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[2] In [7], the authors ask "*Can this type of analysis* [CMA] *be closer to what Nattiez originally thought about the neutrality, objectivity and scientific nature of music analysis? Researchers working in CMA are called to address the issue.*" (pp. 75-76).