A GENERAL THEORY OF EXERGY SAVING
II. ON THE THERMOECONOMIC COST

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Abstract.- In this article the basic idea of exergetic cost (1) is used to theoretically obtain a method for determining the thermoeconomic cost. This method is laid on the clarifying ideas on the F-P representation proposed by Tsatsaronis and Winhold (2) and shows the superstructure of the Thermodynamics-Economics interaction. The two main routes for calculating costs are explained and criticized in the light of the costs hexagon method.

NOMENCLATURE (Other than those in Part I).

Capital latin letters.

$\mathbf{F}$	Fuel exergy vector (nx1) (kW).
$\mathbf{F}^*$	Fuel exergetic costs vector (nx1)(kW).
$\mathbf{P}$	Product exergy vector (nx1)(kW).
$\mathbf{P}^*$	Product exergetic costs vector (nx1) (kW).
$\mathbf{X}$	Represents as a change of basis from system flows to F-P flows.
$\mathbf{Y}^*$	Exergetic amortization costs vector of a system (nx1) (kW).
$\mathcal{Y}^*$	Exergetic amortization and input exergetic costs vector of a system (mx1)(kW).
Z	Economic amortization of a subsystem (mon. units/sec).
$\mathbf{Z}$	Economic amortization cost vector of the subsystems (nx1) (mon. units/sec).
$\mathcal{Z}$	Economic amortization and input thermoeconomic costs vector of a system (mx1) (mon. units/sec).

Small latin letters.

c	Exergoeconomic unit cost of a flow (mon. units /GJ).
$\mathbf{c}$	Exergoeconomic unit cost vector of a system (mx1) or (nx1) (mon. units /GJ).
c^*	Thermoeconomic unit cost of a flow (mon. units /GJ).
$\mathbf{c}^*$	Thermoeconomic unit cost vector of a system (mx1) or (nx1) (mon. units/GJ).

Subscripts.

D	Indicates diagonal matrix.
F	Indicates fuel vector.
P	Indicates product vector.
ω	Indicates thermoeconomic unit cost of input flows of the system (known price of the fuels).

Greek letters.

Δ	Increment.
$\boldsymbol{\kappa}^*$	Exergetic unit cost vector of a system (mx1) or (nx1) (dimensionless).
Π	Thermoeconomic cost of a flow (mon. units/sec).
$\boldsymbol{\Pi}$	Thermoeconomic cost vector of a system (mx1) or (nx1) (mon. units/sec).

Superscripts.

-1	Indicates square matrix inversion
t	Indicates transpose.

1. INTRODUCTION.

Matrix Representation of Exergetic Cost.

In the previous article (1), we defined the exergetic cost, B*, of a physical flow as the amount of exergy needed to produce this flow in a system with prefixed boundaries, an incidence matrix, **A**, which specifies the connections between flows and subsystems and a fuel-product definition which determines the subsystem efficiencies.

Each flow of that system, has a corresponding exergetic cost, B* (kW). From this we got the Exergetic Cost Balance:

$$\mathbf{A} \times \mathbf{B}^* = 0 \tag{1}$$

where $\mathbf{B}^*$ (kW) is the exergetic costs vector, of dimension m. This Balance can also be expressed as:

$$\mathbf{A} \times \mathbf{B}^* + \mathbf{Y}^* = 0 \tag{2}$$

where $\mathbf{Y}^*$ (kW) is the *exergetic amortization* vector, which takes into account the distribution per production time unit of the exergetic cost of each of the subsystems. I.e. the quantity of exergy that was needed to produce each of the subsystems.

In most cases this vector is not considered in cost analysis, and a vector $\mathbf{B}^*$ is obtained which is due only to the *current* destruction of exergy by inefficiencies in the process, and not to that which occurred in the construction of the subsystems or in their maintenance. Thus, henceforth we will use expression (1).

The matrix **A** expresses the physical inputs and outputs to the subsystems but it is not sensitive to our idea of production. Therefore, it is necessary to define (and this is where subjectivity comes in) the concept of product which we have for the whole system, as well as for each of its components.

We get Fuel-Product matrices which satisfy:

$$A = A_F - A_P \qquad (3)$$

Thus, in the example of the previous article (1), we get the matrices A_F and A_P as shown in Tabs. 1 and 2 for its F-P definition of Tab. 1 in (1).

These give the Exergy and Exergetic Cost Balances the following forms:

$$F - P = B_d \qquad (4)$$

$$F^* - P^* = 0 \qquad (5)$$

where

$$P \equiv A_P \times B; \; F \equiv A_F \times B; \; P^* \equiv A_P \times B^*; \; F^* \equiv A_F \times B^* \qquad (6)$$

The vectors **F**,**P** represent the total fuel (kW) which enters each subsystem and the total product (kW) which leaves it, independently of whether or not they are physical inputs or outputs of the subsystem.

In this article the basic idea of exergetic cost is used to show the connection between Thermodynamics and Economics.

To achieve this objective it is necessary to introduce new thermodynamic-economic functions which will be presented in the next section.

The methodology is mainly based on the paper of Tsatsaronis and Whinhold (2) and many ideas follow the approach described by these authors.

Results will be explained with reference to the example, given in Part I (1), at its basic aggregation level, and the F-P definition given by Tab. 4.

This F-P definition does not consider external valuations and so proposition 2R is not applied in the construction of the costs matrix *A* of the system. Therefore we will focus on natural costs.

Because this article is an introductory one, we are mainly interested in natural costs, unless otherwise stated. Therefore, the F-P formulae obtained apply strictly to natural costs. However, it is not difficult to develop similar new formulae for the costs influenced by external valuations.

For example, when the cost of losses is taken to be zero the formulae developed still apply, but the exergy balance will now be:

$$F - P = B_l \qquad (7)$$

where B_l is the vector of exergy destroyed plus the exergy losed by the subsystems.

2. DEFINITIONS.

Given a system with prefixed limits, an aggregation level which specifies its flows and subsystems with prescribed efficiencies, the price of fuels entering the system and the amortization, maintenance and overhead costs of the subsystems, we define:

Thermoeconomic cost, Π, of a physical flow of that system as the quantity of monetary units per sec. requiered to produce this flow.

Unit thermoeconomic cost, c*, of a physical flow of that system as the cost, in monetary units per GJ, of each unit of exergy expended in producing this flow, i.e.:

$$\Pi = c^* \cdot B^* \qquad (8)$$

where c^* [mon. units/GJ], Π [mon. units/sec], B^* [kW].

Unit exergetic cost, κ*, of a physical flow of that system as the amount of exergy expended in producing one exergy unit of this flow, i.e.:

$$B^* = \kappa^* \cdot B, \quad B[\mathrm{kW}], \qquad (9)$$

where κ^* [dimensionless].

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	+1	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	+1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	+1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	+1	-1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	+1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	-1	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0

Tab. 1. Total fuel matrix, A_F, of the system in Fig. 1 and Tab. 4 in Part I.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	0	0	0	+1	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
3	0	-1	+1	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	+1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	+1
8	0	0	0	0	0	0	0	0	0	0	0	0	-1	+1	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	+1	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	+1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	+1	0

Tab. 2. Total product matrix, A_P, of the system in Fig. 1 and Tab. 4 in Part I.

Unit exergoeconomic cost, c, of a physical flow of that system as the cost, in monetary units per GJ, of each unit of exergy of this flow, i.e.:

$$\Pi = c \cdot B, \qquad (10)$$

where c [mon. units/GJ]

Therefore $\qquad c = c^* \cdot \kappa^* \qquad (11)$

We can likewise define this set of costs for the fuel or product flows (virtual or real) of a subsystem which we will introduce subsequently.

3. THERMOECONOMIC COST BALANCE.

In any energy system for which a product, P, and a fuel, F, have been defined, it can be established that: *The thermoeconomic cost of obtaining the product,* P, *is equal to the thermoeconomic cost of the fuel,* F, *used to produce it plus the cost,* Z, *distributed proportionally in time, of the amortization, maintenance and overheads of the production system.* (2).

That is: $\qquad \Pi_P = \Pi_F + Z \qquad (12)$

where $\qquad \Pi_P = c^*_P \cdot P^* = (c^*_P \cdot \kappa^*_P) \cdot P = c_P \cdot P \qquad (13)$

$$\Pi_F = c^*_F \cdot F^* = (c^*_F \cdot \kappa^*_F) \cdot F = c_F \cdot F \qquad (14)$$

and Z [mon. units/sec] can be calculated by the usual procedures for evaluating equipment costs (2,3).

In matrix form, the thermoeconomic costs vector of a system with n subsystems and m flows, defined by an incidence matrix A, is:

$$\Pi = c^*_D \times B^* = B^*_D \times c^* \qquad (m\times 1) \qquad (15)$$

$$\Pi = c_D \times B = B_D \times c \qquad (m\times 1) \qquad (16)$$

$$B^* = \kappa^*_D \times B = B_D \times \kappa^* \qquad (m\times 1) \qquad (17)$$

$$c_D = c^*_D \times \kappa^*_D = \kappa^*_D \times c^*_D \qquad (m\times m) \qquad (18)$$

(The subindex D indicates diagonal matrix).

If the exergetic cost balance, in its most general form, with exergetic amortization, Y*, is:

$$A \times B^* + Y^* = 0 \qquad \text{n equations} \qquad (19)$$

$$\alpha \times B^* - \omega = 0 \qquad \text{m-n equations} \qquad (20)$$

$$\mathcal{A} \times B^* + \mathcal{Y}^* = 0 \qquad \text{m equations} \qquad (21)$$

where

$$\mathcal{A} = \left(\frac{A}{\alpha}\right), \; \mathcal{Y}^* = \left(\frac{Y^*}{-\omega}\right) \qquad (22)$$

then the following analogous expressions can be obtained:

$$A \times \Pi + Z = 0 \qquad \text{n equations} \qquad (23)$$

$$\alpha \times \Pi - [c^*_\omega \cdot \omega] = 0 \qquad \text{m-n equations} \qquad (24)$$

$$\mathcal{A} \times \Pi + \mathcal{Z} = 0 \qquad \text{m equations} \qquad (25)$$

where

$$\mathcal{Z} = \left(\frac{Z}{-c^*_\omega \cdot \omega}\right) \qquad (26)$$

Remember also that the Second Law is expressed as

$$A \times B = B_d \qquad \text{n equations} \qquad (27)$$

Furthermore, since $\mathcal{A}$ is a square matrix which can be inverted, we also get:

$$\kappa^* = -(B)_D^{-1} \times \mathcal{A}^{-1} \times \mathcal{Y}^* \qquad \text{m equations} \qquad (28)$$

$$c^* = -(B^*)_D^{-1} \times \mathcal{A}^{-1} \times \mathcal{Z} \qquad \text{m equations} \qquad (29)$$

$$c = -(B)_D^{-1} \times \mathcal{A}^{-1} \times \mathcal{Z} \qquad \text{m equations} \qquad (30)$$

4. THE COST HEXAGON.

Figure 1 gives the relation between exergy, exergetic cost and thermoeconomic cost and the unit exergetic, exergoeconomic and thermoeconomic costs. It is clear that exergy (which is the minimum possible cost in fuel of a flow), exergetic cost (the true impact on fuel of that flow) and the unit exergetic cost can be obtained from Thermodynamic Analysis. However, the thermoeconomic, unit thermoeconomic and unit exergoeconomic costs must be obtained from the Thermodynamics-Economics interaction.

To solve the basic problem of calculating thermoeconomic costs, it is necessary to go through the following steps:

1. Determine the incidence matrix A(nxm) and the exergy vector of all the flows, **B**, in the system.
2. Define the matrices A_F and A_P for the plant, obtain the matrix α [(m-n)xn] and then $\mathcal{A}$ (mxm) from the F-P-R propositions (1).
3. Calculate the exergetic costs, **B***, by solving the system $\mathcal{A} \times B^* + \mathcal{Y}^* = 0$.
4. Calculate the other exergetic costs, κ^*, F^*, P^*, κ^*_F, κ^*_P.
5. Determine the amortization vector **Z**.
6. Use equations (29) and (30) to calculate the unit economic costs **c** and **c***, and equations (8) or (10) to calculate the thermo-economic cost **Π**.
7. Calculate the thermoeconomic costs of fuel, Π_F, and product, Π_P.
8. Perform a sensitivity analysis to study the influence of any cost with respect to supositions contained in vector $\mathcal{Z}$.

Steps 5 to 8 will be explained later. First we will critically examine, in the light of Fig. 1, the classic methods used to assign costs in a plant.

a) Thermoeconomic route $B^* \rightarrow c^* \rightarrow \Pi$.

This is the most intuitive route and is based on the First Law. It consists of analyzing the impact on fuel of a given flow. Now, taking the unit energetic price of the fuel, and multiplying it by its energetic content, the flow's associated thermoeconomic cost can be obtained.

This procedure is unambiguous and fast when a single product is obtained from one or more fuels, and it can be applied at the maximum aggregation level. However, if we work at lower aggregation levels and there are, or not, residues and by-products, the problem becomes more complicated and the analysis of cost by this technique becomes an art since it is based more on the analyst's judgement and experience than on solid theoretical basis (4,5). Enthalpy-based and non-exergy-based criteria are commonly used.

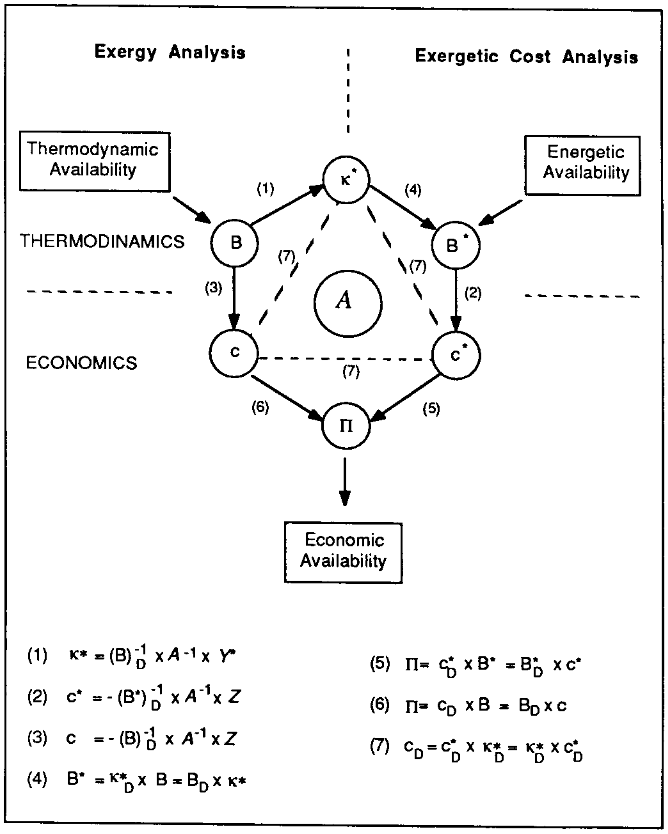

Fig. 1. Cost Hexagon. Shows the costs relationships.

b) Exergoeconomic (2) route B→ c → Π.

A second method, which appeared after the above, for solving the problem of the internal cost distribution, i.e. assigning a cost to each flow in a plant, is based on the Second Law.

Now, the bifurcations are defined by the hypothesis:

$$x = B_j/B_i = (c_j \bullet B_j)/(c_i \bullet B_i) = \Pi_j/\Pi_i \tag{31}$$

where j is the secondary flow and i is the main flow. That is, the unit exergoeconomic costs of the two flows are made equal to each other. However, in a bifurcation there are at least three flows. So what is the criterion for defining the main secondary flows? (6).

In other cases the unit exergoeconomic cost is assigned to a minimum unit cost.

In general, with this method, there is one cost equation per subsystem (a total of "n") and "e" input costs, so "m-n-e" unit cost equations are required whose assignment follows a set of "reasonable" suppositions which are based on adding the "m" necessary equations.

The procedure proposed by El Sayed and Tribus (7) seeks these equations in the distinction, susbystem by subsystem, of thermal, mechanical and chemical irreversibilities. This is quite enlightening and is, in fact, an intuitive approach to the *exergetic cost formation process* described in the previous article (1), because it approaches the systems physical reality in search of the *natural cost*. However, when a subsystem simultaneously has a mixture of different irreversibilities, the definition of cost equations is no longer clear and we again have ambiguities in the allocation.

The fact is that the only external information available is the price of fuel and from this all the costs of flows in any installation must be obtained *deterministically,* from solid theoretical bases. No concept can depend on opinions or theory, and this does not imply that the analyst has no freedom, since the limits of the analysis of the system, the Fuel-Product definition and the exergetic values of residues and/or by-products are set by him/her, but that using logical, preset rules nothing is left to chance.

Therefore, even one questionable supposition regarding concept is enough to invalidate the method since, to obtain all the m thermoeconomic costs, it is necessary to simultaneously solve m interdependent equations. A single bad assignment will lead to more or less bad results in all the other costs, and thus an analysis of sensitivity with respect to the hypotheses will be required.

However, the two methods do have their justification, since they both reveal part of the truth. On the one hand, what determines a product's cost is the cost of fuel and capital costs plus or minus the cost or residues and by-products, and on the other hand, the thermodynamically rational way to distribute costs is via the exergy. The greater the irreversibility the greater the cost. The physics of the problem is what determines the cost.

The synthesis of the above two methods is based -with thermodynamic certainty- on the method proposed in this article. In effect, there is a determinism in the cost calculation process which is based on the uniqueness of *A*. The exergy, exergy losses and our concept of efficiency, are what structurally support the formation of cost. There exist a *natural cost* and a *final cost.* Also, the determinism shapes the calculation method, not the numerical values obtained from the costs. Furthermore, we only need to know the price of the system's input flows, i.e. their unit thermoeconomic cost. With that information, together with the limits of the analysis and the F-P definition, the following set of equations is determined.

$$\boldsymbol{\alpha} \times \boldsymbol{\Pi} - (\mathbf{c}^*_\omega \bullet \boldsymbol{\omega}) = 0 \tag{32}$$

and since we have the equations asociated with each subsystem

$$\mathbf{A} \times \boldsymbol{\Pi} + \mathbf{Z} = 0 \tag{33}$$

we can solve the problem vector **Π** by simply carefully evaluation the vector **Z**.

Moreover, if **Z** is taken to be zero, vector **Π** represents the impact on fuel in monetary units of each and every flow of the system. This, also permits the price of fuels, c^*_ω, entering the system, to be interpreted as an economic weight factor which values the unit of exergy differently depending on the kind of fuel consumed.

If we examine the cost hexagon in Fig. 1, we see that the exergy, B, is the measure of *thermodynamic availability* of a flow; the exergetic cost, B*, can be associated with a *energetic availability* of that flow since its definition includes such factors as the production efficiency, the amount of fuel required to produce it and the energy required to manufacture the subsystem which produces the flow. It is clear that the lower the exergetic cost, the greater the energetic availability. One example of this is water heating using solar panels. Independently of the thermodynamic availability of solar energy, to use it is necessary to consume exergy in manufacturing the panel. Depending on the panel design, this exergy may be greater than would be collected by the panel during its working life. It is clear that the energetic availability of solar energy is related to its exergetic cost, and not to its exergy. On the other hand, *economic availability* which depends both on the fuels and their prices and on the cost of the subsystems producing the flow. The lower the thermoeconomic cost, the greater the economic availability of the product. Returning to the solar panel example, an industrial process which lowers panel production costs will increase their economic availability.

3. THE VECTOR Z.

The elements of the vector **Z** of dimension m are, according to equation (26).

n values of the type Z_j

m-n values of the type $-c^*_\omega \bullet \omega$

The first values correspond to the capital invested, and the maintenance and overheads associated with the subsystem j. As with the exergetic cost of the subsystems, their value depends on the limits of analysis and on the analyst's objectives. Tsatsaronis and Winhold (2) give a useful methodology to calculate them.

The m-n remaining elements of the vector Z take the following values depending on the type of flow:

a) If it is an input flow to the system, then $Z_i = -c^*_i \bullet \omega_i$. Normally, if the system inputs are fuels or electrical energy, its exergetic cost, has the same numerical value, ω_i, as its exergy and these, in turn, may be equal to its energetic content, and c^*_i will be the corresponding fuel price.

b) If the flow is a residue, then $Z_r = c^*_r \bullet \omega_r + Z_r$. That is, its thermoeconomic cost will include the amount of fuel required to dispose of the residue and the weighted annual cost of the disposal installation.

c) If the flow is a by-product, then $Z_r = c^*_r \bullet \omega_r + Z_r$. The thermoeconomic value of a by-product is the thermoeconomic cost of the best really-available alternative use. This will have amortization and maintenance cost, Z_r, and a price, c^*_r, of the fuel saved by using the by-product.

d) If the flow is an internal recycle, then $Z_i = 0$, since there are no alternative uses other that recycle, once we accept that the system has a structure defined by A.

4. THERMODYNAMIC COSTS OF THE F-P REPRESENTATION.

Once **A, B, F, P, B*, Z, Π, c*** and **c** have been determined and calculated, the thermodynamic costs of fuel and product can be calculated. For this, we vectorially define:

$$\mathbf{\Pi_F \equiv A_F \times \Pi, \quad \Pi_P \equiv A_P \times \Pi} \tag{34}$$

therefore

$$\mathbf{\Pi_F - \Pi_P + Z = 0} \tag{35}$$

To calculate the F-P expressions of the unit costs, we will follow the two routes in Fig. 1.

a) Exergoeconomic route ($\mathbf{B \rightarrow c \rightarrow \Pi}$).

We vectorially define:

$$\mathbf{c_F \equiv F_D^{-1} \times \Pi_F, \quad c_P \equiv P_D^{-1} \times \Pi_P} \tag{36}$$

Therefore

$$\mathbf{c_F = F_D^{-1} \times A_F \times \Pi, \quad c_P = P_D^{-1} \times A_P \times \Pi} \tag{37}$$

which give the unit exergoeconomic costs in the F-P representation. Also, the cost increase, due to production, for the whole system is represented in matrix form as follows:

$$\begin{aligned} \mathbf{\Delta c} &= \mathbf{c_P - c_F = F_D^{-1} \times [Z+(B_d)_D \times c_P] =} \\ &= \mathbf{P_D^{-1} \times [Z+(B_d)_D \times c_F]} \end{aligned} \tag{38}$$

This vector expression coincides with that given in scalar form for this route by Tsatsaronis and Winhold (2). Furthermore, $\mathbf{c_F}$ and $\mathbf{c_P}$, of dimension n, are related to the vector **c**, of dimension m, by:

$$\mathbf{c_F = X_F \times c \quad ; \quad c_P = X_P \times c} \tag{39}$$

where **X** represents a change of basis which has the form:

$$\mathbf{X_F \equiv F_D^{-1} \times A_F \times B_D \quad ; \quad X_P \equiv P_D^{-1} \times A_P \times B_D} \tag{40}$$

b) Thermoeconomic route ($\mathbf{B^* \rightarrow c^* \rightarrow \Pi}$).

We define, in vector form:

$$\mathbf{c^*_F \equiv (F^*)_D^{-1} \times \Pi_F \, , \quad c^*_P \equiv (P^*)_D^{-1} \times \Pi_P} \tag{41}$$

Therefore

$$\mathbf{c^*_F = (F^*)_D^{-1} \times A_F \times \Pi, \quad c^*_P = (P^*)_D^{-1} \times A_P \times \Pi} \tag{42}$$

which give the unit thermoeconomic costs in the F-P representation. It is also of interest to evaluate the unit thermoeconomic costs for the installation:

$$\begin{aligned} \mathbf{\Delta c^*} &= \mathbf{c^*_P - c^*_F = (F^*)_D^{-1} \times [Z - Y^*_D \times c^*_P] =} \\ &= \mathbf{(P^*)_D^{-1} \times (Z - Y^*_D \times c^*_P]} \end{aligned} \tag{43}$$

If, as is usual in exergetic cost analyses, the exergetic amortization **Y*** is not included, the above expression has the following form:

$$\mathbf{\Delta c^* = c^*_P - c^*_F = (F^*)_D^{-1} \times Z = (P^*)_D^{-1} \times Z} \tag{44}$$

which indicates that, for a given system, the unit thermoeconomic cost increase (mon.units/GJ of fuel used) is proportional to the term Z only, in contrast with expression (38) which is explicit in Z and in B_d.

It is again clear that if the amount of fuel required to produce a product is known, or guessed at, and the term Z is evaluated, then exergetic analysis is not necessary in order to obtain the cost of the product. This is fully true at the maximum aggregation level. However, if we wish to understand the origin of the economic cost, we must go down to the most micrososcopic aggregation level and separate the effect of exergy destruction from the amortization, maintenance, etc... costs, Z, in each particular subsystem. The exergoeconomic factor proposed by Tsatsaronis and Winhold (2) studies these effects.

7. SENSITIVITY ANALYSIS.

Since the matrix A and the vector **B** are determined by the system's own characteristics (once the F-P structure has been defined), then all the cost variations must be produced by the suppositions made in calculating **Z**. Thus, if the superscript, t, indicates transpose, we have:

$$\partial \mathbf{c}/\partial \mathbf{Z}^t = -\partial[\mathbf{B_D}^{-1} \times A^{-1} \times \mathbf{Z}\,]\,/\partial \mathbf{Z}^t = -\mathbf{B_D}^{-1} \times A^{-1} \tag{45}$$

In the same way, we get:

$$\begin{aligned} \partial \mathbf{c_F}/\partial \mathbf{Z}^t &= \partial \mathbf{c_F}/\partial \mathbf{c}^t \bullet \partial \mathbf{c}/\partial \mathbf{Z}^t = \partial[\mathbf{X_F} \times \mathbf{c}]/\partial \mathbf{c}^t \bullet \partial \mathbf{c}/\partial \mathbf{Z}^t \\ &= -\mathbf{F_D}^{-1} \times \mathbf{A_F} \times A^{-1} \end{aligned} \tag{46}$$

$$\partial \mathbf{c_F}/\partial \mathbf{Z}^t = -\,\mathbf{P_D}^{-1} \times \mathbf{A_P} \times A^{-1} \tag{47}$$

Also

$$\partial \mathbf{c^*}/\partial \mathbf{Z}^t = -\,(\mathbf{B^*})_\mathbf{D}^{-1} \times A^{-1} \tag{48}$$

$$\partial \mathbf{c^*_F}/\partial \mathbf{Z}^t = -\,(\mathbf{F^*})_\mathbf{D}^{-1} \times \mathbf{A_F} \times A^{-1} \tag{49}$$

$$\partial \mathbf{c^*_P}/\partial \mathbf{Z}^t = -\,(\mathbf{P^*})_\mathbf{D}^{-1} \times \mathbf{A_P} \times A^{-1} \tag{50}$$

From these expressions, it is clearly simple to study the influence of any cost with respect to the suppositions contained in the vector Z. Also, one very important observation is that the physical structure of the problem, i.e. **A**, $\mathbf{A_P}$, $\mathbf{A_F}$ and **B**, contain all the information on sensitivity!.

8. RESULTS AND CONCLUSIONS.

As an example of the results of calculating thermoeconomic costs, take the hypothetical case in Fig. 1, with F-P definition from Tab. 4 in Part I. This is a greatly simplified example, since our aim here is to present a method and describe some qualitative tendencies, and not to respond quantitatively a physical reality.

Thus, Tab. 3 shows the assumed levelized capital, operating and mainteinance costs of the 30 MW plant, i.e. 6400000 \$/year with 6000 operating hours per year. Furthermore, the unit thermoeconomic cost, c^*_1 (or c_1), of coal is taken to be 3.5385 \$/GJ, while that of air is taken to be zero.

Also, Tab. 4 gives the levelized costs, Z_i, of the subsystems making up the plant. The distribution of these values is aproximately similar to that given by Tsatsaronis and Winhold (8) for a 1500 MW plant. These values, together with the thermoeconomic costs of coal and air from Tab. 3, are the only economic data requiered to solve the problem of costs, once the costs matrix, A, and the flow exergy vector **B** have been defined.

Capital costs (10^6 \$/year) : 4.178
Operating and maintenance costs (10^6 \$/year) : 2.222

Total operating costs (10^6 \$/year) : 6.400
Service hours (hours/year) : 6000.

Unit thermoeconomic cost of coal, c^*_1, (\$/GJ) : 3.5385
Unit thermoeconomic cost of air, c^*_2, (\$/GJ) : 0.0000

Thermoeconomic cost of coal, $\Pi_1 = c^*_1 \bullet B^*_1$, (¢/s) : 34.9000

Thermoeconomic cost of air, $\Pi_2 = c^*_2 \bullet B^*_2$, (¢/s) : 0.0000

Tab. 3. Economic assumptions to solve the costing problem of the plant in Fig. 1 in Part. I.

Denomination	No.	Z_l (¢/s)	%
Combustion Chamber	1	5.926	20.00
Steam Generator	2	13.600	45.90
Air-Gas Preheater	3	1.185	4.00
Boiler-Turbine Steam Pipe	4	0.012	0.04
High Pressure Turbine	5	5.227	17.64
Low Pressure Turbine	6	1.600	5.40
Condenser	7	1.191	4.02
Feedwater Pump	8	0.172	0.58
Heat Exchanger	9	0.415	1.40
Bifurcation Element	10	0.012	0.04
Generator	11	0.237	0.80
Electricity Distributor	12	0.053	0.18
Boiler	13	20.711	69.90
Steam Cycle	14	8.919	30.10
Power Plant	15	29.630	100.00

Tab.4. Assumed levelized capital, maintenance and operating costs of the equipment of the plant in Fig. 1 in Part I.

Tabs. 5, 6, 7 give the results of applying equations (21), (25), (15), (16) and derived equations.

It is observed that the variation of the unit exergoeconomic cost for the different flows in plant is very high. However, the unit thermoeconomic and exergetic costs are more stable throughout the production process. This last statement is of fundamental importance, since one procedure which has been quite widely used up to now, where equations were lacking to solve the cost problem, consists of setting the unit exergoeconomic costs of some flows equal to each other. Because of the variability of these costs, this could leave to substancial errors.

Formula (11): $c = \kappa^* \cdot c^*$, is the key, since it states that every flow is composed of two multiplicative components: a purely thermodynamic one, whose value depends on the irreversibilities existing in the production process up to the arisal of the flow, and a purely economic one, whose value depends on fuel costs and investsments required to produce the flow. These ideas will be analyzed in detail in Part III.

Thus, Thermodynamics gives us:

1. A structure, $\boldsymbol{A}$, for calculating costs, which extends into the field of Economics, and which, however, contains some subjective information, i.e. our concept of efficiency, and
2. A function, κ^*, the unit exergetic cost, whose value is independent of economic considerations, but which is the basis of costing, and can be utilized to objectively compare different installations, i.e. the step by step efficiency of energy use.

Flow No.	Exergy B(kW)	Exergetic Cost B^*(kW)	Unit Exergetic Cost κ^* (no dim)	Unit Thermoeconomic Cost c^*($/GJ)	Unit Exergoeconomic Cost c($/GJ)	Thermoeconomic Cost Π(¢/s)
1	98 630	98 630.0 *	1.000	3.539	3.539	34.9000
2	11	11.0 *	1.000	0.000	0.000	0.0000
3	1 665	2 937.4	1.764	5.324	9.393	1.5639
4	1 973	2 713.0	1.375	4.112	5.654	1.1155
5	71 891	98 854.6	1.375	4.112	5.654	40.6444
6	6 995	9 618.6	1.375	4.112	5.654	3.9547
7	3 782	6 691.9	1.769	5.344	9.455	3.5759
8	47 441	104 016.0	2.193	5.746	12.598	59.7671
9	46 781	104 016.0	2.223	5.747	12.779	59.7791
10	19 323	42 964.0	2.223	5.747	12.779	24.6919
11	14 184	31 537.7	2.223	5.750	12.785	18.1338
12	3 581	7 962.2	2.223	5.750	12.785	4.5782
13	72	1 568.2	21.780	7.220	157.251	1.1322
14	609	3 493.6	5.737	7.365	42.247	2.5729
15	4 377	14 779.9	3.377	6.412	21.653	9.4774
16	5 139	11 426.4	2.223	5.750	12.785	6.5701
17	63	140.1	2.223	5.749	12.784	0.0805
18	22 870	61 051.9	2.670	6.603	17.628	40.3142
19	31 470	84 627.4	2.689	6.555	17.626	55.4698
20	29 580	84 627.4	2.861	6.583	18.833	55.7068
21	673	1 925.4	2.861	6.589	18.851	1.2686
22	28 907	82 701.9	2.861	6.589	18.851	54.4912
23	300	6 534.1	21.780	7.220	157.251	4.7175

Tab. 5. Hexagon costs results of the system's flows of Fig. 1 and Tab. 4 in Part I.

Flow No.	Exergy F(kW)	Exergetic Cost F*(kW)	Unit Exergetic Cost κ^*_F	Unit Thermoeconomic Cost c^*_F (\$/GJ)	Unit Exergoeconomic Cost c_F (\$/GJ)	Thermoeconomic Cost Π_F (¢/s)
1	100 295	101 567.4	1.013	3.590	3.636	36.4639
2	64 896	89 236.0	1.375	4.112	5.654	36.6897
3	6 995	9 618.6	1.375	4.112	5.654	3.9547
4	47 441	104 016.0	2.193	5.746	12.598	59.7671
5	27 458	61 052.0	2.223	5.747	12.777	35.0872
6	10 603	23 575.5	2.223	5.750	12.783	13.5556
7	3.644	8 102.3	2.223	5.750	12.783	4.6587
8	673	1 925.4	2.861	6.589	18.851	1.2686
9	5 076	11 286.3	2.223	5.750	12.783	6.4896
10	19 323	42 964.0	2.223	5.747	12.776	24.6919
11	31 470	84 627.4	2.689	6.555	17.626	55.4698
12	29 580	84 627.4	2.861	6.583	18.833	55.7068

Tab. 6. Hexagon fuel costs results of the subsystems of Fig. 1 and Tab. 4 in Part I.

Flow No.	Exergy P(kW)	Exergetic Cost P*(kW)	Unit Exergetic Cost κ^*_P	Unit Thermoeconomic Cost c^*_P (\$/GJ)	Unit Exergoeconomic Cost c_P (\$/GJ)	Thermoeconomic Cost Π_P (¢/s)
1	73 864	101 567.6	1.375	4.112	5.654	41.7599
2	43 064	89 236.1	2.072	5.636	11.678	50.2897
3	5 436	9 618.3	1.769	5.343	9.453	5.1389
4	46 781	104 016.0	2.223	5.747	12.778	59.7791
5	22 870	61 051.9	2.670	6.603	17.628	40.3142
6	8 600	23 575.5	2.741	6.429	17.622	15.1556
7	372	8 102.3	21.780	7.220	157.251	5.8497
8	537	1 925.4	3.585	7.483	26.827	1.4407
9	3 768	11 286.3	2.995	6.118	18.324	6.9045
10	19 323	42 964.1	2.223	5.750	12.783	24.7039
11	29 580	84 627.4	2.861	6.583	18.833	55.7068
12	29 580	84 627.4	2.861	6.589	18.851	55.7598

Tab. 7. Hexagon product costs results of the subsystems of Fig. 1 and Tab. 4 in Part I.

Note, finally, that in our example the thermoeconomic cost of fuel, Π_i=34.9 ¢/s, is always greater (in some cases by one or two orders of magnitude) than the value of Z_i. Thus, among the components of vector **Z** in equation (25), the economic term associated with fuel dominates. That is the exergetic cost, exergetic expense or impact on fuel function, B*, or its unit function, κ^*, is dominant in the optimization and synthesis of the system's cost structure, to the extent that costs are allocated by applying propositions 2F, 2P and/or 2R based on that function.

One question remains: Will these functions be satisfied when exergy expense is not the dominant term in calculating costs?...

Information is a property to which the concept of exergy (9) and, with it, that of exergetic cost can be extended. The door is open for further research.

Furthermore, there is another, perhaps more universal property namely "production of products" or simply "production", which is broken down into various materials, exergy, information, money, etc. depending on the subject of analysis. Does "production" satisfy the above propositions?. This is another open door for further research. The answer might be found in work done in the field of industrial management with regard to production optimization and simplification (synthesis) and to cost engineering.

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