MULTIPLY TWO NEGATIVE NUMBERS IS NEGATIVE NUMBER. DIVISION POSITIVE NUMBERS, NEGATIVE NUMBERS, POSITIVE AND NEGATIVE NUMBERS AND NEGATIVE AND POSITIVE NUMBERS CAN BE POSITIVE NUMBER, NEGATIVE NUMBER OR ZERO.

It is time that we raise above simplified understanding of multiplication and division

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Abstract: In this scientific work is proved that multiply two negative numbers is negative numberand that division positive numbers, negative numbers, positive and negative numbers and negative and positive numbers can be positive number, negative number or zero.

Keywords: Multiplication, Division, Positive, Negative, Numbers

Introduction: In primary school we learnt that result of multiplication two negative numbers is positive number and that result of division two negative numbers is positive number. In this work is proved that what we learnt was not correct.

MULTIPLICATION NUMBERS

Area is defined as set of dots of a plane. We shall mark set of dots of plane with G. Characteristics of plane known from planimetry are:

1. Plane is not negative number

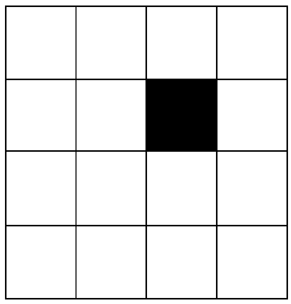
2. If $G_1=G_2$, then $P(G_1)=P(G_2)$. With letter P is marked area of polygons G_1 and G_2.

3. If $G_1 \cap G_2 = 0$, then $P(G_1 \cup G_2)=P(G_1)+P(G_2)$.

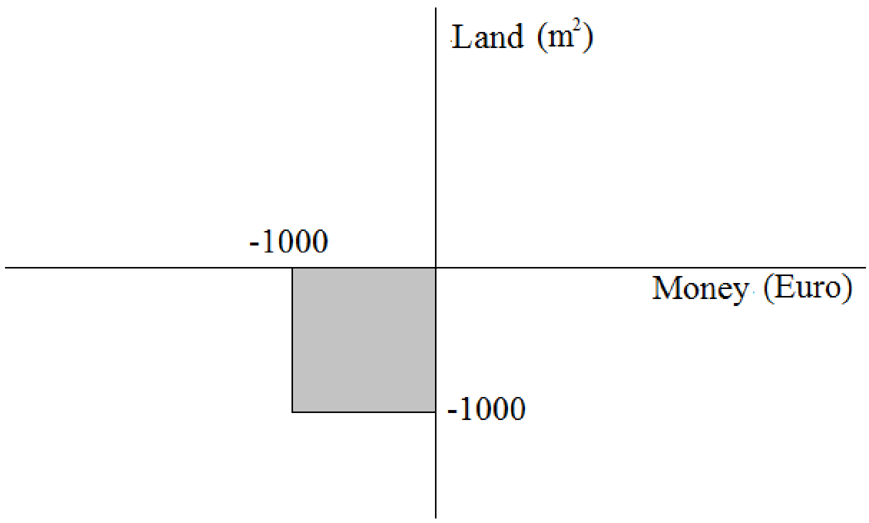
Area can be negative. We shall show it on the three following examples.

1. If we have a square divided by horizontal and vertical lines on smaller squares and if one of that small squares is missing (black little square on the figure). Assume that little squares are made of paper. Then, if we want to fill the missing little square, we

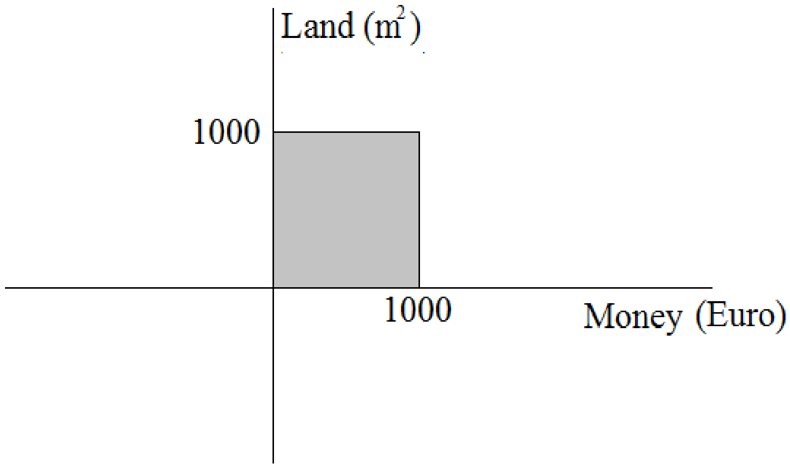
should take paper, cut the little square and glue it on the place where the little square is missing. That area was obviously negative before we glued the missing little square.



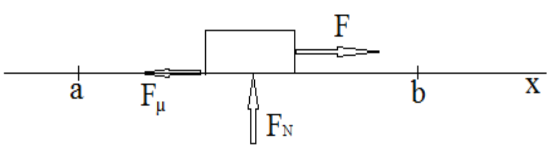
2.Some man wants to plant wheet, but he has not got money to buy seed nor the land to plant seed. He decides to borrow 1000 Euros and to borrow a land for planting seed. If we show that in Descartes's coordinating system it will be



To reach his goal this man had to borrow money and land and he doesn't own nor money nor land. That area shown in the third quadrant of the Descartes's coordinating system cannot be positive or not negative. That area will be positive only if it will be in the first quadrant of the coordinating system.



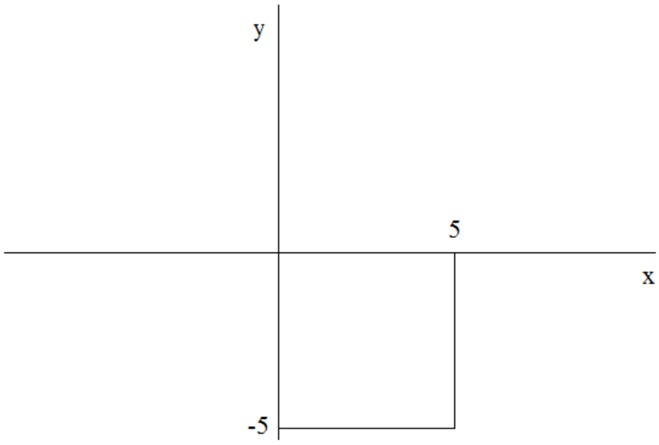
3. The best example of negative area is negative work. If we have object A and force F which moves that object from point a to point b and if we have the force of sliding friction $F\mu = \mu F_N$, then the total work is



$$W=\int_a^b F\,dx - \int_a^b F_\mu\,dx$$

Definite integral from a to b Fµdx is negative area because it makes moving of the object A more difficult.

Now have a look on the fourth quadrant of the coordinating system.



This area is bordered by directions x=0 , x=5 , y=0 and y= -5. This area is negative.

$$G_4 = \int_0^5 -5dx = -5x\Big|_0^5 = -25$$

Area in the first quadrant of the coordinating system is positive and has the same absolute value as the area in the fourth quadrant.

$$G_1 = \int_0^5 5dx = 5x\Big|_0^5 = 25$$

If we add area from the first quadrant to area from the fourth quadrant, the result should be zero.

$$G_1 + G_4 = 25 - 25 = 0$$

But if we apply the rule for definite integral, that lower border must be smaller than higher border and if we add two mentioned areas, we shall have:

$$x=\int_{-5}^{5} 5dy=5y|_{-5}^{5}=25-(-25)=50$$

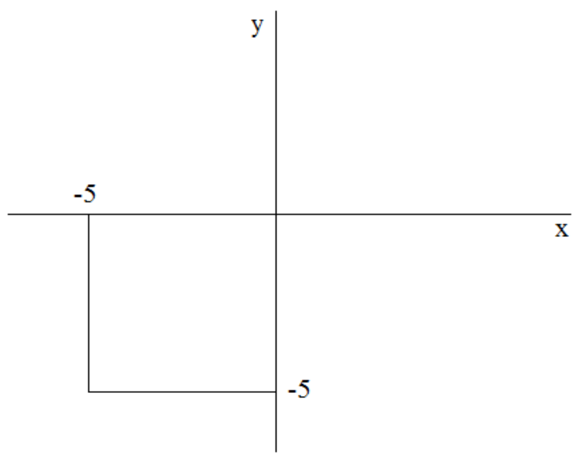
This means that, if we consider that negative area exists, something is wrong. If we set borders of definite integral as follows

$$x=\int_{0}^{5} 5dy+\int_{0}^{-5} 5dy$$

it will be

$$x=25-25=0$$

If we apply that on the third quadrant of the coordinating system



it will be

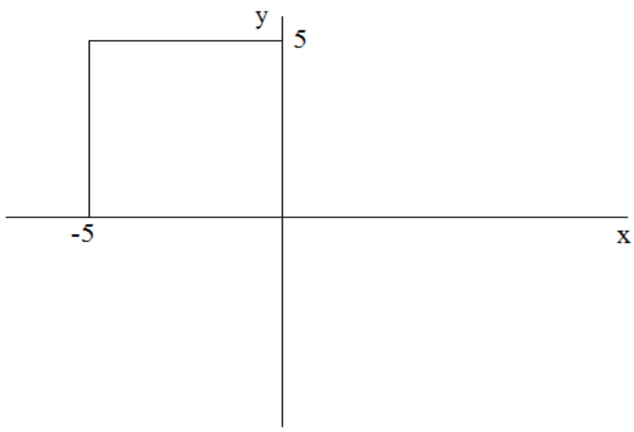
$$y=\int_{0}^{-5} -5dx= -5x|_{0}^{-5}=(-5)\ (-5)$$

According to our previous knowledge result of this multiplication is +25. But that area is negative because it is made of set of negative numbers (elements). Any dot that we pick from that set will be negative. Because area is set of dots of a plane and because all elements of that set are negative numbers, then that area is also negative. From that results that value of mentioned area is

$$y=(-5)\cdot(-5)= -25$$

This means that multiplication two negative numbers is negative number.

Let's look second quadrant of the coordinating system.



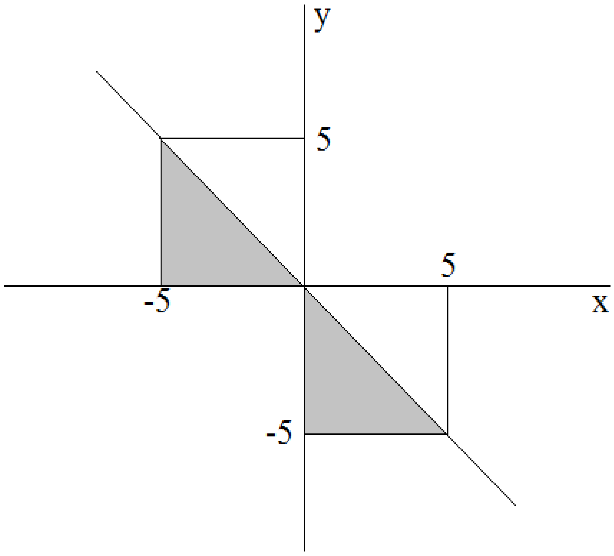
$$y=\int_{0}^{-5} 5dx=5x\Big|_{0}^{-5}=5\cdot(-5)= -25$$

Area in the second quadrant of the coordinating system is also negative.

It should be added that area in the first quadrant of the coordinating system is absolute positive because it contains only positive numbers. Area in the third quadrant of the coordinating system is absolute negative because it contains only negative numbers.

Areas in the second and fourth quadrant of the coordinating system are negative, but these are sets of dots of planes which contain positive and negative numbers. We shall show that parts of that areas can be more or less positive.

If we draw direction y= -x through areas in the second and fourth quadrant of the coordinating system



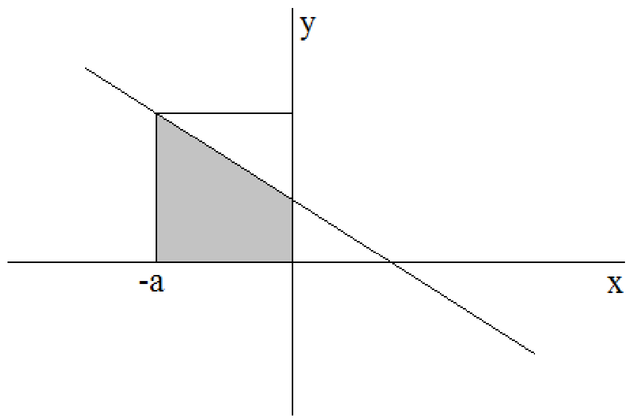
we shall see that area bordered by directions y= -x, y=0 and x= -5 is positive.

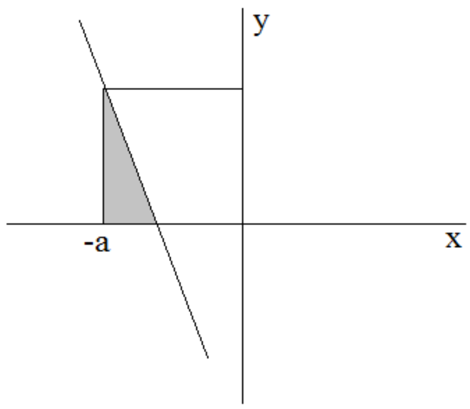
$$y=\int_0^{-5} -x\,dx = -x^2/2\Big|_0^{-5} = -[(-5)\cdot(-5)] = -(-25)/2 = 12{,}5$$

In the fourth quadrant is area bordered by directions y= -x, x=0 and y= -5 also positive.

$$x=\int_0^{-5} -y\,dy = -y^2/2\Big|_0^{-5} = -(-5)\cdot(-5)/2 = 12{,}5$$

If we draw any direction y= -kx±n through the second quadrant, area bordered by negative part of x direction, positive part of y direction or zero of function, direction y= -kx±n and any direction x= -a, will be positive.



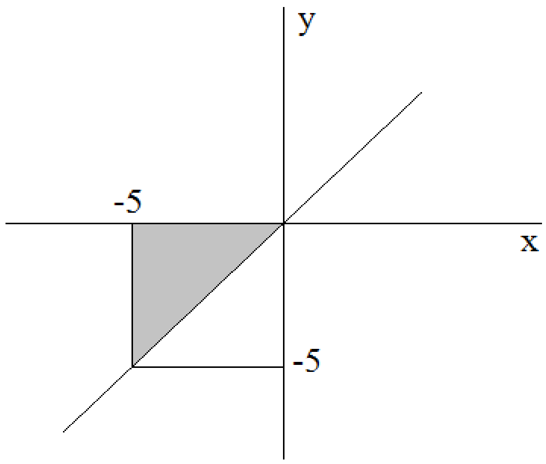


The same is in the fourth quadrant, only it should be taken into consideration that direction
y= -kx±n and negative part of y direction border that area.

This means that areas in the second and in the fourth quadrant are sometimes negative and sometimes partly positive and partly negative. That suits to the character of that areas, because they contain positive and negative numbers.

If we do the same with sine and cosine functions in the fourth quadrant we shall see that area bordered by sine function is negative and area bordered by cosine function is positive.

For the area in the third quadrant and direction y=x is



$$y=\int_{0}^{-5} x dx = x^2/2|_{0}^{-5} = (-5)^2/2 = -12,5$$

This area is negative. Any area bordered by direction y=kx ± n which goes through that area, direction x= -a, negative part of y direction or zero of function and negative part of x direction is also negative.

Consequences of above mentioned are:

Until now we counted that multiplication two negative numbers is positive number and that square root from negative number is imaginary number. We should make corrections as follows:

a) If we have

$$y^2= -25$$

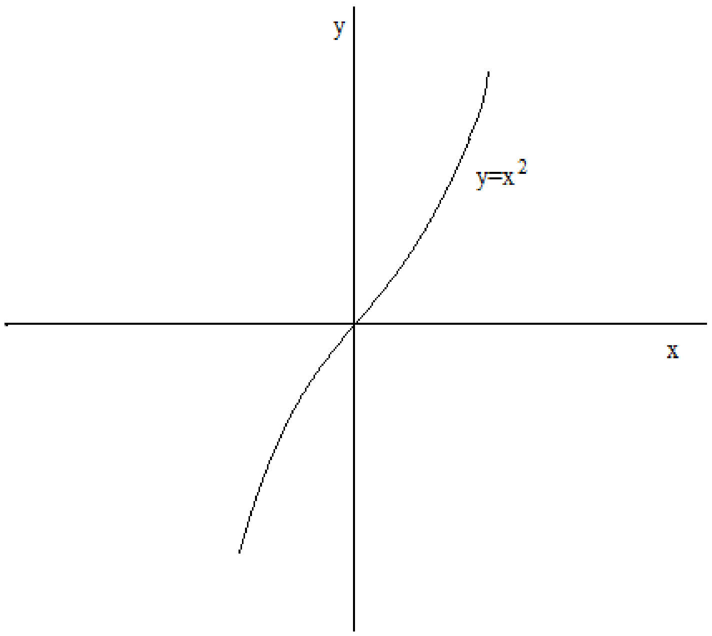
we should take into consideration where that y^2 came from. If y^2 came from multiplication two positive numbers y·y then it is imaginary number. But if y came from y·(-y) or (-y)·y or (-y)·(-y) then it is not imaginary number, because $-y^2= -25$, that is -y= -5. This shows that square with side y belongs to the third quadrant.

b) If we want to find zeros of function

$$\pm ax^2 \pm bx \pm c$$

and if discriminant D<0.

It should be said that this function does not have shape of previous curve, than looks like more to the curve $y=x^3$ and it has only one zero.



c) Circle equation can be $x^2+y^2=r^2$ but it can be also $\pm x^2 \pm y^2 = \pm r^2$. Is it going to be sign + or - depends on origin of square of x,y and r as it is explained in a).

This means that circle exists out of its until now known domain. If we take into consideration that square root from negative number is real number (except if that square root belongs to the first quadrant), then there is not limitation and equation $\pm x^2 \pm y^2 = \pm r^2$ exists for any x.

Things that we see using our eyes is not maybe that it really looks like. If we look from
three-dimensional space and using senses adjusted for three-dimensional things, than it can be only projection of multi-dimensional space in three-dimensional space. For example projection of ellipse from three-dimensional space in two-dimensional space can be circle. Projection of ellipsoid from four-dimensional space in three-dimensional space can be circle, ellipse or sphere. Their projection in two-

dimensional plane can be circle or ellipse.

Circle equations would be:

For I quadrant: $x^2+y^2=r^2$ ($x>0$, $y>0$, $r>0$)

For II quadrant: $y^2-x^2=r^2$ ($x<0$, $y>0$, $r>0$)

For III quadrant: $y^2+x^2= -r^2$ ($x<0$, $y<0$, $r>0$)

For IV quadrant: $y^2-x^2= -r^2$ ($x>0$, $y<0$, $r>0$)

Instead one, circle has now four equations.

If radius of a circle is negative.

For I quadrant: $x^2+y^2= -r^2$

For II quadrant: $y^2-x^2= -r^2$

For III quadrant: $-y^2-x^2= -r^2$

For IV quadrant: $-y^2+x^2= -r^2$

This is a negative circle. If area can be negative, why it could not be a line. If the negative area can be the missing area (example 1), then circle can be the missing line.

If we look in the circle equations for $r<0$ and $r>0$ we shall see that the circle equation for the first quadrant for $r<0$ is same as the circle equation for the third quadrant for $r>0$. The same thing is for the second and the fourth quadrant. That means that coordinate system is rotated for 180 degrees and the circle for $r<0$ is like reflection in the mirror of the circle for $r>0$.

d) If line and area can be negative, then it can be a volume, as missing space. Such space can be negative in three-dimensional bodies as well as in multi-dimensional bodies.

e) Any number and not only square numbers which can be a result of multiplication of two or more numbers can be corrected.

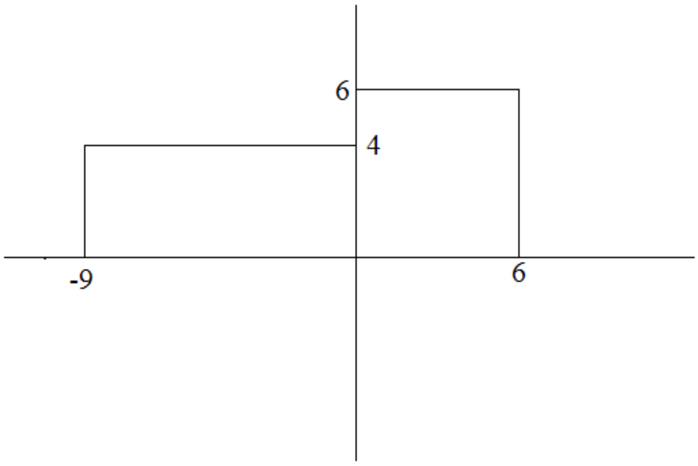
DIVISION NUMBERS

If we apply division for II, III and IV quadrant we come to very interesting conclusion. If negative area from the second and the fourth quadrant is divided with negative number, result is positive number. But if negative area from the third quadrant is divided with negative number, the result is negative number. That means that result of division of two negative numbers can be a negative but it can also be a positive number.

If mentioned characteristics of negative and positive areas can be applied for square, then they also can be applied for rectangle, which represents multiplication of any two different numbers.

If area from the first quadrant is divided with the value on the abscissa or the ordinate from the first quadrant, the result will be positive number

36:6=6



If positive area from the first quadrant is divided with value on abscissa of area from the second quadrant, the result will be value on the ordinate from the second quadrant.

36:(-9)=4

If the same area is divided with value on ordinate of area from the second quadrant, the result will be value on the abscissa

36:4= -9

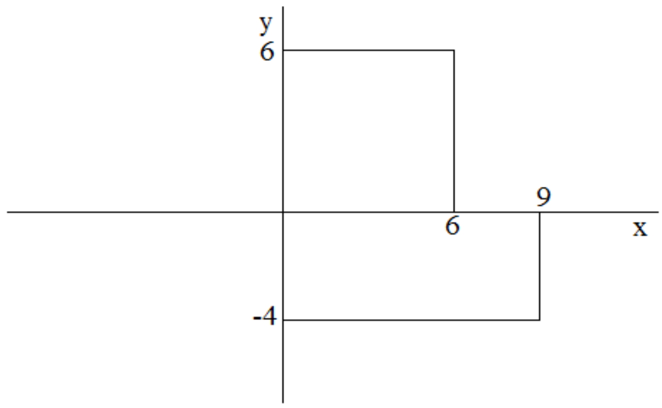
If we do the same with the area from third quadrant

36:(-9)= -4

36:(-4)= -9

but we must remember that (-4)·(-9) is not 36.

If area from the first quadrant is divided with value on the abscissa from the fourth quadrant



36:9= -4,

and with value on the ordinate

36:(-4)=9

For result of division and multiplication is important in which quadrant division and multiplication occur.

Area in the first quadrant can be shown as six men and each one has six Euros and that is area of 36 Europeople.

Area in the second quadrant can be shown as four men and each of them has debt of nine Euros, that is four of them have debt of 36 Euros. That is area of -36 Europeople.

For areas in third and fourth quadrant we should understand what people on negative part of ordinate mean. These are people which are not able to get money. Not able means that it is not possible to get money to them. That impossibility can be for example:

- Some man owes to four people four Euros. His debt is counted for him and for his creditors as -4 Euros. When he earns, borrow or get that money, his money condition is not anymore -4 Euros then +4 Euros. But the people to which he owes have to go to trip, after some accident they end up on deserted island or on some place unknown to their debtor. That is now situation 4:(-4). Now can be following cases:

  a) Debtor is not able to pay to his creditors (those four people are for example on deserted island). The result of division in this case is -1, because those four people cannot get their money.

  b) If those four people went in some town and they need four Euros to come back in their town, then they can borrow four Euros, because they count that their debtor will give them four Euros when they come back in their town. Now the result of division is 4:(-4)=1 because they got their money back.

- One more example of division with negative number is: Negative number is (are) person(s) that do(es) not exist. If some man decides to give four Euros to not existing persons or to beings to which money is pointless like animals, plants or inhabitants of other planets, then the result of division is (if he decided to give money to four of that beings) 4:(-4)=0. Someone can say that four animals got 1 Euro each and that is true but if we take into account that that money is pointless for them, then it is clear that the result of division is zero. If it would be food for animals, then the result of division would be different.

Let's look now what can be, besides debt, negative value on abscissa.

- If we would like to give four Euros that we do not have, that is -4 Euros, to four existing persons, that means if we would like to divide (-4):4. Following cases can occur:

  a) If we promised to give one Euro to each person, then they expected and counted on that one Euro, The result of division in this case is (-4):4= -1, because they did not get their money.

  b) If we did not promise to give them one Euro each, then they do not expect that money and result of division is zero, because they did not get money and their state, counting on money that they could get is zero.

  c) And at the end, if we do not have 4 Euros and we want to divide them on four people. We promised to those people that we shall give them 1 Euro each and we have good chances for getting that money. Then those people can borrow 4 Euros counting that we shall their debt give back. They were given 1 Euro each and the result of division is (-4):4=1.

If six people from the I quadrant give their money to four debtors from the II quadrant, then we have

$$36:(-9)=4,$$

that is six people from the first quadrant should give to four people 9 Euros each to give their debt back.

But if we observe area in the second quadrant, which means debt, then

$$(-36):(-9)=4,$$

that is four people have debt 9 Euros each.

If we do the same in the III quadrant, then positive area divided to debt

$$36:(-9)= -4,$$

because area in the III quadrant represents debtors but they are not able to get or give money from one of the reasons explained above.

If we divide absolute negative area from the III quadrant to value on the abscissa of absolute positive area from the I quadrant, the result of division will be value on the positive part of the ordinate.

$$(-36) :6=6$$

This is debt of the people which are not able to give their debt back. Their debt should give back six people which are able to do it.

By analogy we can conclude all other cases.

Conclusion of above mentioned is:

Result of division two positive, two negative, positive and negative or negative and positive numbers can be positive number, negative number or zero.

Division area to area

Division area to area comes from division area to number. If we for example positive area from the first quadrant want to divide to negative area from the second quadrant, then it is

$G_1:G_2=36:[(-9)\cdot 4]=4$ men : 4 men =1