Valence band spin splitting in strained \( \text{In}_{0.18}\text{Ga}_{0.82}\text{As/GaAs quantum wells} \)

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Valence band spin splitting in strained In$_{0.18}$Ga$_{0.82}$As/GaAs quantum wells

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Abstract. Magneto-transport at low temperature has been employed to study a series of p-type In$_{0.18}$Ga$_{0.82}$As/GaAs quantum wells with carrier concentrations in the range $(1.5-4.3) \times 10^{11}$ cm$^{-2}$. Tilting the field from the growth direction does not change the relative magnitudes of the spin-split conductivity minima. This is a remarkable effect, and confirms the recent results of Martin et al on an In$_{0.15}$Ga$_{0.85}$Sb/GaSb quantum well. Strain decoupling of the $|M_J|=\frac{3}{2}$ and $|M_J|=\frac{5}{2}$ states projects the spin onto the growth direction. We find here that this effect occurs for the $\nu = 3, 4$ and $\nu = 5, 6$ and $\nu = 7, 8$ spin-split conductivity minima. The Shubnikov–de Haas oscillations in perpendicular field have been compared to the Landau levels calculated with an eight band $k \cdot p$ model and this determines an upper limit to the value of $\kappa_f$, the spin–magnetic-field coupling parameter.

The presence of strain in quantum wells can completely change the nature of the valence band structure. In a recent paper Martin et al. [1] have shown that the valence band spin develops a two-dimensional character, while a number of authors have shown how the hole mass can be reduced by factors of up to four [2–5]. These changes can be used to enhance potential device performance as discussed by O’Reilly [2]. Structures in which the well material is under compression in the interface plane have been most commonly studied, and in this case the ‘heavy hole’ band edge lies above the ‘light hole’ band edge, typically by ~ 80 meV for a lattice mismatch of 1%. This energy splitting is much larger than the Fermi energy in a p-doped system. The $|M_J|=\frac{3}{2}$ decoupling causes the effective mass for in-plane motion for a lattice mismatch of 1% to become light [2–5]. In addition the decoupling causes the ‘heavy hole’ spin to be projected along the growth direction such that $M_J$ is a good quantum number independent of the orientation of an applied magnetic field. This was observed recently by Martin et al [1] on an In$_{0.15}$Ga$_{0.85}$Sb/GaSb p-type quantum well, showing that the valence band spin has a completely two-dimensional character. In this paper we report a low-temperature magneto-transport study of a series of p-type In$_{0.18}$Ga$_{0.82}$As/GaAs quantum wells. By tilting the field with respect to the growth direction the novel spin behaviour is observed over a range of carrier concentrations $p_s$ and filling factors $\nu$ ($\nu = \hbar p_s/eB$). The relative strengths of the Shubnikov–de Haas oscillations in a perpendicular field are very sensitive to the value of the spin–magnetic-field coupling parameter $\kappa_f$. A comparison of our experimental results with a $k \cdot p$ theory calculation of the Landau levels suggests that $\kappa_f \leq 1.8$ for the In$_{0.18}$Ga$_{0.82}$As alloy.

p-type In$_x$Ga$_{1-x}$As/GaAs quantum wells have been investigated previously. Temperature-dependent Shubnikov–de Haas experiments [3] first revealed the existence of a light in-plane effective mass, and this has been subsequently confirmed by direct cyclotron resonance experiments [6–8]. The totally decoupled limit is not achieved and similar results have been published on In$_{0.15}$Ga$_{0.85}$Sb/GaSb quantum wells [5]. The strain reconstruction of the valence band has been shown to affect the electrical properties right up to room temperature [9]. The ‘heavy hole’ band is non-parabolic, and the non-parabolicity contribution to the effective mass has been shown to scale inversely with the ‘heavy’ hole–‘light hole’ splitting [10]. A pressure dependence of the valence band structure has also been inferred from Hall measurements [7]. The range of samples available here has allowed us to investigate the magneto-transport properties as a function of $p_s$, or equivalently, as a function of the filling factor $\nu$. Magneto-optical properties of these samples will be published elsewhere [8]. In this paper we concentrate particularly on the spin properties revealed in the quantum transport. Previously published magneto-transport data [3, 6] of this system do not resolve any spin splittings in the Shubnikov–de Haas oscillations.

The samples were grown by MBE at the RSRE Laboratory...
in Malvern. Four samples were studied each with indium concentration 18% and 90 Å well width. The strain in the interface (x, y) plane is \( \varepsilon_{xx} = \varepsilon_{yy} = -1.35\% \) for this composition, and the well width is lower than the critical thickness of \( \approx 100 \) Å expected from the results of Fritz et al [11]. The samples were p-type modulation doped with spacer layers of 150 Å and the doped layers were doped at \( \approx 5 \times 10^{17} \) cm\(^{-3} \). The carrier concentration was controlled by the width of the doped layers on either side of the well. The lowest low-temperature concentration of holes in the quantum well was \( p_h = 1.5 \times 10^{11} \) cm\(^{-2} \) (sample ME541) and the highest was \( p_h = 4.2 \times 10^{11} \) cm\(^{-2} \). For samples ME541 and ME539, the two with the lowest doping, \( p_h \) could be significantly increased by a persistent photoconductivity effect. The sample the two with the lowest doping, ME541 and ME539, are listed in table 1. Mobilities were in the 5000-15 000 cm\(^2\)(V s)\(^{-1}\) range. Contacts were made by alloying In/Zn (= 5% Zn) into the material, processed to either a van der Pauw or Hall bar geometry. Magneto-transport experiments were performed with fields either up to 9 T in a resistive magnet or up to 14.5 T in a superconducting magnet, and temperatures between 0.35 and 4.2 K. The sample could be rotated with respect to the field with a pumped \(^4\)He system.

Table 1. The details of the In\(_{0.16}\)Ga\(_{0.84}\)As/GaAs quantum wells. Each quantum well is 90 Å wide, with 150 Å spacer layers between the quantum wells and two p-type layers, doped at \( \approx 5 \times 10^{17} \) cm\(^{-3} \). Layer thicknesses \( L_A \) (\( L_B \)) is the thickness of the doped region on the substrate (cap) side of the quantum well. \( p_h \) is the low-temperature carrier concentration both before (dark) and after (light) illumination.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( L_A ) (Å)</th>
<th>( L_B ) (Å)</th>
<th>Dark ( p_h ) (( \times 10^{11} ) cm(^{-2} ))</th>
<th>Light ( p_h ) (( \times 10^{11} ) cm(^{-2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME541</td>
<td>0</td>
<td>75</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>ME539</td>
<td>30</td>
<td>75</td>
<td>2.6</td>
<td>3.7</td>
</tr>
<tr>
<td>ME538</td>
<td>30</td>
<td>125</td>
<td>3.7</td>
<td>4.0</td>
</tr>
<tr>
<td>ME540</td>
<td>125</td>
<td>125</td>
<td>4.2</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Figure 1 shows magneto-transport data for sample ME539 (\( p_h = 2.6 \times 10^{11} \) cm\(^{-2} \)) and for sample ME540 (\( p_h = 4.2 \times 10^{11} \) cm\(^{-2} \)) before illumination. Well resolved conductivity minima are observed for fields such that \( \nu = 1 \) is integral, the well known Shubnikov–de Haas effect. For low \( \nu \) the conductivity gets very close to zero over a wide field range. Note particularly the \( \nu = 1 \) minimum for ME539 which extends over a \( \nu \sim 4.8 \) T range. Corresponding behaviour is seen in the Hall voltage. Figure 1 shows clean quantum Hall plateaux at \( \nu = 2, 3, 4, 6 \) and weaker features at \( \nu = 5 \) and 7 for ME540. Reliable Hall data could only be obtained from the Hall bar samples as problems with the contacts gave anomalous structure in the van der Pauw geometry. The Shubnikov–de Haas oscillations were very similar in both cases however. This is why the Hall data for sample ME539 is not shown in figure 1 as a van der Pauw geometry was used for this particular experiment. Even at 0.5 T the low-field oscillations are smeared out by \( kT^* \) broadening. At 0.35 K we could observe oscillations down to 0.27 T for sample ME540 which corresponds to filling factors up to \( \nu = 60 \). This illustrates the high quality of the samples. The quantum transport is quite similar in structure to that observed from n-type systems, and is noticeably simpler than that recorded from p-type GaAs/Al\(_{x}\)Ga\(_{1-x}\)As heterostructures [12, 13]. These experiments were carried out on p-type heterojunctions in which the situation is complicated by the much greater amount of heavy hole-light hole mixing and the lifting of the spin degeneracy in the subband dispersion even at zero field by the combined effect of the large spin–orbit interaction and lack of inversion symmetry. Ando [14] has calculated theoretically the band structure of p-type GaAs/Al\(_{x}\)Ga\(_{1-x}\)As heterojunctions and quantum wells. The reason for the relatively simple behaviour observed here is that the valence band Landau levels are considerably simplified by the strain decoupling of the \( |M| = \frac{3}{2}, \frac{5}{2} \) states. These states otherwise interact strongly (via off-diagonal elements in the effective Hamiltonian) and give rise to very complicated Landau levels and hence complicated quantum transport.

The Landau levels in perpendicular field have been calculated to compare the valence band structure with our experimental results. We use an eight-band \( k \cdot p \) Hamiltonian [15, 16] in a flat band approximation with continuity of envelope function and envelope function 'current' at the interfaces [17]. Strain is included with the valence band Hamiltonian of Pikus and Bir [18] and a hydrostatic contribution for the two conduction band diagonal terms. The axial approximation (replacement of \( \gamma_2 \) and \( \gamma_3 \) by an average, in this case \( \frac{1}{2} (\gamma_2 + \gamma_3) \)) in \( H_{23}, H_{34}, H_{45}, H_{56}, H_{67}, H_{78} \) of the Hamiltonian \( \hat{H} \) has been used for a compact numerical evaluation of the solutions. The input parameters to the model are listed in table 2. The GaAs masses of Shanabrook et al [19] are used, and are taken to vary with indium concentration \( x \) [20] as

\[
m_{hh}^{01} = 0.34
\]

\[
m_{hh}^{10} = 0.0042 - 0.062x
\]

\[
m_{hh}^{11} = 0.725
\]
in units of the free electron mass. The heavy hole well depth is taken to be 69.2 meV which is 33% of the difference between the GaAs and the strained In_{0.18}Ga_{0.82}As band gaps. This offset ratio has been used in an extensive study of In_{0.18}Ga_{0.82}AsGaAs (z \leq 0.15) quantum wells and superlattices by Duggan and co-workers [20]. This offset gives a band gap for the quantum well studied here that agrees with the measured band gap to within ~10 meV. The Landau levels are numbered as n_v = -2, -1, 0, ..., in a standard notation [21]. The Landau levels calculated consist essentially of two harmonic series, one with n_v = -2, -1, 0, ..., and one with n_v = 1, 2, 3, .... The n_v = -2 state has pure M_f = -\frac{3}{2} character. To discuss the nature of the other levels originating at B = 0 from the 'heavy hole' subband edge we consider only the admixtures of the J = \frac{3}{2} valence band states, the 'heavy hole' M_f = \pm \frac{3}{2}, and 'light hole' M_f = \pm \frac{1}{2} basis functions as the spin-orbit and conduction bands are relatively distant in energy from the 'heavy hole' subband edge. The n_v = -1 state is an admixture of the M_f = -\frac{5}{2} and M_f = -\frac{3}{2} eigenstates, and the n_v = 0 state has a further admixture of the M_f = \pm \frac{1}{2} basis state. The higher-order solutions n_v = 1, 2, ..., involve all the basis states in the effective Hamiltonian, although the off-diagonal term between the M_f = -\frac{3}{2} and M_f = +\frac{3}{2} basis functions is zero. The solutions of interest here lie within the Fermi energy, 6.4 meV for p_F = 4.2 \times 10^{11} cm^{-2}, at the top of the first 'heavy hole' subband whereas the barrier light hole band edge lies 61.0 meV lower than the 'heavy hole' subband edge, and the well 'light hole' band edge lies 7.2 meV lower still (the system is type-II for light holes). Thus admixture of the |M_f\rangle = \frac{3}{2} and |M_f\rangle = \frac{1}{2} states is small because of the strain. The two harmonic series in the Landau fan diagram thus have M_f = -\frac{5}{2} (n_v = -2, -1, 0, ...) and M_f = \pm \frac{3}{2} (n_v = 1, 2, ...), and the pairs of states with opposite spin (n_v = -2, 1; -1, 2; 0, 3; ...) are spin split by an amount dependent on the Luttinger parameter \kappa. The totally decoupled limit is not achieved in these samples, as deduced from cyclotron resonance experiments in which the mass is \approx 0.16, heavier than the totally decoupled limit of 0.10. The residual interactions must then be included to model the valence band, and this we do with the effective Hamiltonian approach. We note also that the energy difference between the first two heavy hole subbands is 23.6 meV, large compared to the Fermi energy and so anti-crossing effects between Landau levels of the same n_v originating from different band edge energies are irrelevant here. The Landau level structure is therefore very simple. The heavy hole–light hole interactions that lead to very complicated valence bands in wide unstrained quasi-two-dimensional systems have been removed by the strain, and multiple subband occupation is not possible.

The experimental results (figure 1) show that the strengths of the conductivity minima are greater for even \nu than for odd \nu. For example in figure 1(b) the \nu = 5 feature is weak compared to the neighbouring \nu = 4 and \nu = 6, \nu = 7 is weak, \nu = 9 is not properly resolved, and at lower field only the even series can be observed. To make a comparison of this behaviour with the Landau level calculations we assume that the magnitude of a conductivity minimum at some particular (integral) \nu depends on the energy discontinuity of the \nu = 0 Fermi energy at the appropriate field. The relative magnitudes of adjacent minima can be compared in this way. We are able to estimate \kappa for the In_{0.18}Ga_{0.82}As alloy because the relative strengths of the Shubnikov–de Haas minima are very sensitive to the value of \kappa assumed. This process is illustrated in figure 2 which shows the valence band Landau levels for three different \kappa. \kappa = 2.37 is the value linearly interpolated between the GaAs and InAs values [22] (1.2, 7.68 respectively), and predicts a Shubnikov–de Haas series that is dominated by the odd series (figure 2(c)). This is clearly too large a spin splitting. The other extreme comes from taking \kappa = 1.2, the GaAs value. The Landau levels in this case (figure 2(a)) have a very small spin splitting. The actual \kappa lies somewhere between the two extremes, and we suggest that \kappa = 1.8 (figure 2(b)) is a reasonable upper limit if we interpret the experimental results in a one-particle way. However, the presence of the carriers will increase the spin splitting through the exchange enhancement, a many-body effect. This was first observed by Fang and Stiles [23] in Si n-type inversion layers, and has subsequently been studied in detail for both Si and GaAs systems [24, 25]. The enhancement is different for different Landau levels, and Ando and Uemura [26] first pointed out that the spin splitting should be an oscillatory function of Landau level filling. At low fields the spin-split Landau levels overlap and the exchange enhancement is modest. Conversely at very high field the exchange enhancement can be very large [25] and ultimately saturates when the spin states are completely separated [25]. It is difficult to quantify the magnitude of this effect here. Measurements of the exchange enhancement in n-type systems [24, 25] can be made with the so-called ‘coincidence method’. This technique depends on the two-dimensional nature of the orbital motion and the three-dimensional nature of the spin so that different spin-split Landau levels become degenerate as the sample is rotated with respect to the field. The main point of this paper is that the ‘heavy hole’ spin in strained quantum wells has two-dimensional character and so there are no such degeneracies on rotating the sample. In other words, rotating the field gives no extra information about the magnitude of \kappa. Spin splittings can also be measured from activation plots of the conductivity \sigma_{xx} [25]. We have not performed such experiments because of two complications. Firstly, the exchange interaction itself may increase with decreasing temperature as the polarization

### Table 2. The parameters used in the Landau level calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GaAs</th>
<th>In_{0.18}Ga_{0.82}As</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band gap (eV)</td>
<td>1.519</td>
<td>1.2574</td>
</tr>
<tr>
<td>Spin–orbit splitting (eV)</td>
<td>0.341</td>
<td>0.329</td>
</tr>
<tr>
<td>Kane matrix element E_k (eV)</td>
<td>25.7</td>
<td>25.7</td>
</tr>
<tr>
<td>Conduction band mass</td>
<td>0.0665</td>
<td>0.05867</td>
</tr>
<tr>
<td>Heavy hole mass [001]</td>
<td>0.340</td>
<td>0.340</td>
</tr>
<tr>
<td>Heavy hole mass [111]</td>
<td>0.725</td>
<td>0.725</td>
</tr>
<tr>
<td>Light hole mass [001]</td>
<td>0.0942</td>
<td>0.0830</td>
</tr>
<tr>
<td>Lattice constant (Å)</td>
<td>5.65</td>
<td>5.727</td>
</tr>
<tr>
<td>Elastic constant C_{11} (kbar)</td>
<td>1223</td>
<td>1153</td>
</tr>
<tr>
<td>Elastic constant C_{12} (kbar)</td>
<td>571.1</td>
<td>550</td>
</tr>
<tr>
<td>Deformation potential a (eV)</td>
<td>-7.1</td>
<td>-6.88</td>
</tr>
<tr>
<td>Deformation potential b (eV)</td>
<td>-1.7</td>
<td>-1.72</td>
</tr>
</tbody>
</table>

Valence band spin splitting in strained quantum wells
of the carriers into different spin states increases. Secondly, the technique tends to underestimate the splitting because of the spread of extended states at the centre of each Landau level.

It is clear that the exchange interaction can only increase the spin splitting and hence the comparison of the Shubnikov-de Haas data with the Landau level calculations gives an upper estimate to $\kappa_I$. The upper limit of $\kappa_I = 1.8$ lies below the linearly interpolated 2.37, suggesting considerable bowing in the $\kappa_I$ versus indium concentration relation. Our results suggest

$$\kappa_I(z) \leq \kappa_I(0) + (1 - z)(\kappa_I(1) - \kappa_I(0)) - cz(1 - z),$$

where $z$ is the indium concentration, $\kappa_I(0) = 1.2$, $\kappa_I(1) = 7.68$ [22] and $c = 3.8$.

The more remarkable behaviour comes when the sample is rotated, by angle $\theta$, with respect to the magnetic field. The experimental results are shown in figure 3 for $p_e = 4.2 \times 10^{11}$ cm$^{-2}$ (sample ME540). The entire trace simply scales as $\cos \theta$. The angle as measured from the Shubnikov-de Haas oscillations, assuming that $p_e$ is fixed and that the level degeneracy is determined only by the perpendicular component of field, agreed with the rotation gear setting to within the accuracy of the rotation gear. For the low-field minima where we do not resolve any spin splitting this behaviour is conventional for two-dimensional systems. However we also observe $\theta$ dependence of the magnitudes of the spin-split minima. The data for ME541 and ME539 are plotted in figure 4. In this case the magneto-resistance is plotted against the perpendicular component of field showing how the form of the trace is independent of the tilt angle. Specifically we have been able to study the $\nu = 3,4$ minima for $p_e = 1.8, 2.6, 3.7$ and $4.2 \times 10^{11}$ cm$^{-2}$ (ME541 illuminated, ME539 dark, ME539 illuminated and ME540 dark); the $\nu = 5,6$ minima for $p_e = 3.7$ and $4.2 \times 10^{11}$ cm$^{-2}$ (ME539 illuminated and ME540 dark); and the $\nu = 7,8$ minima for $p_e = 4.2 \times 10^{11}$ cm$^{-2}$ (sample ME540), although the $\nu = 7$ minimum is not completely resolved at 1.5 K. The amplitudes are independent of $\theta$ to within our experimental uncertainty of $\sim 5\%$. The $\nu = 5,6$ minima for $p_e = 4.2 \times 10^{11}$ cm$^{-2}$ provide an extreme example—at $\theta = 64.9^\circ$, $\nu = 5$ appears at ($B_1, B_2$) = (7.2 T, 3.4 T) and $\nu = 6$ appears at ($B_1, B_2$) = (6.1 T, 2.8 T) ($B_0$ is the field in the $(x,y)$ plane and $B_z$ is the field along the $z$ axis, the growth direction.) We emphasize that tilting conventional n-type two-dimensional systems [23-25] gives very different results. In the n-type case tilting the field from $\theta = 0$ causes the Landau level spacing to follow $\cos \theta$ whereas the spin splitting remains the same as it is determined by the total field; orbital motion is quantized in two dimensions, the spin is not. In this strained layer sample it would appear then that the spin is also quantized into a two-dimensional behaviour as well as the orbital motion. This is precisely the behaviour observed by Martin et al. [1] in the In$_{0.15}$Ga$_{0.85}$Sb/GaSb case. In the In$_{0.45}$Ga$_{0.55}$Sb/GaSb system the spin splittings dominate over the cyclotron energies for wide wells giving an odd Shubnikov-de Haas series. Our results show that the effect also occurs in a system in which the spin splitting does not dominate the Landau level dispersion, and we have demonstrated that it occurs over a wide range of $\nu$.

The explanation of this spin effect depends on the strain decoupling of the $|M_j\rangle = \frac{3}{2}, \frac{1}{2}$ states. An examination of the Hamiltonian [1, 15, 16] for $B = (B_x, B_y, B_z)$ shows that the element between the $M_j = \pm \frac{3}{2}$ and $M_j = \pm \frac{1}{2}$ basis states is zero, and the diagonal terms include $\pm \frac{1}{2} \kappa I B_z$.

---

**Figure 2.** The valence band Landau levels for a 90 Å In$_{0.15}$Ga$_{0.85}$As/GaAs quantum well calculated using an eight-band $\kappa \cdot \pmb{p}$ method with the parameters listed in table 2. $\kappa_I = 1.2$ for GaAs and three different $\kappa_I$ for the In$_{0.15}$Ga$_{0.85}$As alloy. ($\kappa_I$ is the Luttinger spin-magnetic-field coupling parameter.) (a) $\kappa_I = 1.2$ is the GaAs value; (b) $\kappa_I = 1.8$ is the value that we suggest best reproduces the strengths of the experimental Shubnikov-de Haas oscillations (figure 1) if the exchange interaction is neglected; (c) $\kappa_I = 2.37$ is linearly interpolated between the GaAs and InAs values. The energy zero is at the top of the heavy hole quantum well.
for $M_f = \pm \frac{3}{2}$. The spin splitting in the decoupled limit then depends only on $B_z$. Equivalently, $M_f$ remains a good quantum number independent of $B$ providing that the $|M_f| = \frac{3}{2}, \frac{1}{2}$ states are decoupled. The strain provides the axis of quantization, not the applied magnetic field. This is the basic observation that explains the effect. The explanation is not influenced by any exchange interactions. The effect also depends on the large spin–orbit coupling which makes the interactions between the spin–orbit $J = \frac{1}{2}$, $|M_f| = \frac{1}{2}$ states and the $J = \frac{3}{2}$, $|M_f| = \frac{3}{2}$ states unimportant. However we know that with respect to the cyclotron mass the decoupled limit is not achieved in our samples. In fact the heavy-hole–light-hole decoupling could not be increased very much in the In$_{0.16}$Ga$_{0.84}$As/GaAs system beyond the case studied here, because of critical thickness considerations. The non-zero $B_x$ and $B_y$ add terms to the Hamiltonian for $B = (0, 0, B_z)$. Firstly, the diagonal terms are altered, either through $k_z$ or the number operator $(a_+a_- + \frac{1}{2})$ depending on the gauge. These terms give rise to a diamagnetic shift [27] and do not directly contribute to a spin splitting. Secondly, new off-diagonal terms appear coupling the heavy hole and light hole states. The point is that the off-diagonal terms do not significantly alter the spin splitting over the range of $B$ used here, and so the bands behave as though they are decoupled in the tilted field magneto-transport experiment. At $|B| = 9$ $T$ and $\theta = 70^\circ$ the $B_z$ component of field gives a spin splitting that is at least an order of magnitude larger than the contribution to the spin splitting from the $B_{xy}$ field estimated from second-order perturbation theory [1]. Ultimately, the off-diagonal terms will couple the $|M_f| = \frac{3}{2}, \frac{1}{2}$ states, the spin splitting will depend on the field orientation, and the simplicity of the decoupled picture will be lost [28]. To observe these effects one could study the present samples to much higher fields and at lower temperatures so that higher $\nu$ spin-split conduction minima could be observed, or one could study samples in which the heavy hole–light hole splitting is lower.

To conclude, we report the results of magneto-transport experiments on p-type strained In$_{0.16}$Ga$_{0.84}$As/GaAs quantum wells. Clear quantum transport is observed. In a tilted field the spin is shown to be aligned along the growth direction independent of the angle of tilt for our experimental extremes: $B_x/B_y < 2.8$ and $(B_{xy}^2 + B_z)^{\frac{1}{2}} < 9$ $T$. This result comes from the observation that the entire Shubnikov–de Haas trace simply scales as $\cos \theta$. These results confirm the earlier observation of Martin et al [1] on an In$_{0.18}$Ga$_{0.82}$Sb/GaSb quantum well, and arise as a consequence of uniaxial strain heavy hole–light hole decoupling. This is an interesting example of anisotropy caused by a perturbation which differentiates between different directions in the crystal. An analysis of the $\theta = 0$ spin-split Shubnikov–de Haas oscillations with a calculation of the valence band Landau levels allows us to determine an upper limit to $\kappa_1$, the spin–magnetic-field coupling parameter.

References