Similarity Laws of Re-Entry Aerodynamics - Analysis of Reverse Flow Shock and Wake Flow Thermal Inversion Phenomena

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Summary. A computational fluid dynamics (CFD) based dimensional analysis of the flow in the base region of a planetary re-entry configuration is presented. Reynolds number and free stream Mach number are found to have the dominant influence on the Mach number of the trapped recirculating flow in the wake. Prandtl number is found have the strongest influence on the temperature of the wake recirculating flow. Two associated flow phenomena, wake reverse flow shock (WRFS) and wake flow thermal inversion (WFTI) are introduced. The governing role of the Prandtl number on the wake flow energy budget and thus the base region heating is discussed.

1 Introduction

The present work uses computational fluid dynamics (CFD) to investigate the steady-state, perfect gas fluid dynamics of a planetary re-entry capsule at zero angle of attack with the emphasis on the flow physics of the base region. The study employs non-dimensional similarity parameters to generalize its findings. The fluid dynamics phenomena of the fore-body flow have been well documented for many years. Knowledge of the base flow aerodynamics, however, remains sparse. The present work is motivated by the need to understand the physics of the base flow to provide knowledge for re-entry spacecraft design optimisation and risk reduction processes.

Dimensional analyses of the Navier-Stokes equations show that the Reynolds number (Re), Prandtl number (Pr), ratio of heat capacty (γ) and Mach number (M) form a set of pi groups suitable for the study of the re-entry aerodynamics with adiabatic walls. With the aid of these similarity parameters, this document investigates two phenomena associated with the flow in the base region, the wake reverse flow shock (WRFS) and wake flow thermal inversion (WFTI). The former has been observed before in numerical studies (for example, [2]) but never in experiments. The latter is, to our knowledge, a new phenomenon. A CFD based investigation of the dependence of the maximum reverse flow Mach number on the Reynolds number and the free stream Mach number is presented. The effect of the Reynolds number on the maximum reverse flow Mach number and recovered base stagnation temperature is found to be qualitatively similar to that of the free stream Mach number. The energy budget for the trapped wake recirculation flow is discussed. The Prandtl number is proposed as the primary determining parameter between the balance of energy of work done on the trapped recirculation region via viscous interactions and heat transfer out of it by thermal diffusion. The dependence of the base stagnation point temperature on the Prandtl number is documented.

The prototype re-entry configuration considered is that of the Beagle II mission to Mars, the demise of which provided the motivation for the present work. The approach adopted in this study is to perturb the similarity parameters around a selected datum point to reveal the flow physics. The datum planetary atmosphere is modelled as an ideal gas with properties corresponding to CO_2. The datum free stream conditions are those of a point at $26km$ altitude on a typical entry trajectory through the Martian atmosphere. The static temperature used is $192K$, the static pressure is $54Pa$ and the velocity is $4000ms^{-1}$. The Mach number is 18.19 for γ 1.333 and the free stream Re_D is 544270.

The CFD code used here is the commercial code CFD^{++} [3]. CFD^{++} can solve both the steady or unsteady compressible Navier-Stokes equations, including multi-species and finite-rate chemistry modelling. For the present work, the calculations performed are double precision and of second-order accuracy in both time and space. Concerning the spatial discretization, total variation diminishing (TVD) polynomial interpolation with MinMod limiting is used, while an implicit Runge-Kutta method is used for the time integration. A structured 2 dimensional mesh is used in this study. The mesh blocking topology is an O-grid adjacent to the body geometry surrounded by a C-grid, which captures the wake flow. The steady-state, ideal gas CFD computations are performed on a 2-dimensional axi-symmetric model of the external shape of the Beagle II's thermal protection system with minor modifications. The CFD did not model the backward facing step cut at the aft side of the shoulder nor the launch clamp ring at the base [4].

2 Flow Conditions

One of the purposes of the present work is to investigate the effects of similarity parameters on wake flows of different scales. The CFD simulations are conducted using dimensional values for the gas properties with modifications to adjust the relevant non-dimensional quantities. Therefore, the far-field or inlet boundary conditions for the CFD simulations are required in the form of velocity (U), temperature (T) and pressure (P). The present study employs the total enthalpy and Mach number to specify the-far field fluid dynamics at the chosen trajectory point, thus requiring a transformation between the two sets of quantities. The enthalpy equation and the definition of the Mach number provide two linear equations in U^2 and T with a unique solution for velocity and temperature. Given free stream Mach number M_∞, total enthalpy and γ, the free stream velocity and temperature can be obtained by solving these two equations. The free stream temperature serves as a constraint to the relationship between the free stream pressure and density via the ideal gas law. Given a prescribed inlet temperature, a desired free stream density can then be obtained, to achieve to a desired Reynolds number, by adjusting the inlet pressure. We shall call this method of changing the free stream Reynolds number to obtain similar solutions the *density adjustment method*. Scaling the reference viscosity μ_0 and conductivity k_0 of the Sutherland law allows matching of the Reynolds and Prandtl numbers for flows with different body length scales. We shall call this method of obtaining similar solutions *viscosity adjustment method.*

3 Similarity

Laminar, ideal gas (CO_2), steady-state CFD simulations are used here in the investigation of similarity parameters. A study of generating similar solutions with density method is first conducted. Meshes for both full scale Beagle II and 1/10 scale version are constructed and CFD simulations are carried out with the free stream pressure and density ten times that of the datum values for the 1/10 case. The other gas parameters and the free stream conditions retain their datum point values. The converged solutions show agreements typically within 0.2% between the two length scales. The solution Mach number, temperature and flow speed are compared in figures 1. and 2. The body length scales are matched during the post processing for the purpose of comparison in the diagrams below. Other parameter fields examined in the study include total temperature, total pressure, viscosity, thermal conductivity and Prandtl number fields.

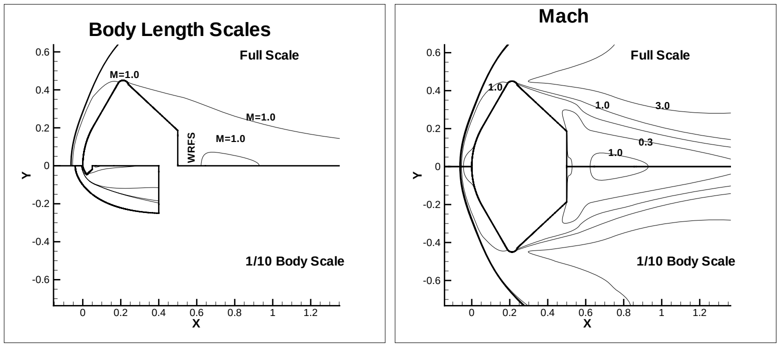

Fig. 1. Left- The 1/10 scale solution (bottom half) made similar to full scale model (top half)using density adjustment method in true spatial scale. Right- The 1/10 model expanded by x 10 during post processing. Mach number solution fields are plotted. Note the reverse flow supersonic region. WRFS occures as flow leaves $M = 1$ region towards the base of the probe.

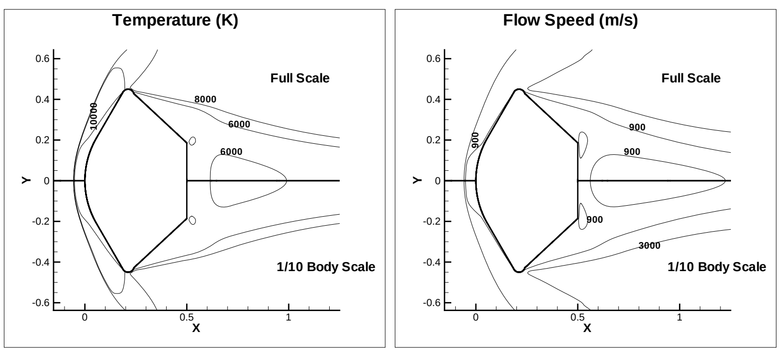

Fig. 2. Left- The temperature fields. Right-The velocity fields. Top half is full scale model and the bottom half is 1/10 scale similar solution matched with density adjustment method

The 1/10 scale solutions are also shown to be similar to the full scale solution via the viscosity adjustment method which necessitates decreasing μ_0 by ten times. Here k_0 is decreased 10 times in order to match the Prandtl number. Similar agreements as for the density adjustment method are observed.

By using both the density and viscosity adjustment methods, several selected values of γ and Mach number solutions have been matched for 1/10th body length scale models with that of the full-scale model. The solution at our selected datum point shows a Wake Reverse Flow Shock. In the solutions of the 1/10 model with the datum far-field conditions, without any matching of the similarity parameters, the WRFS vanishes. The overall velocities in the latter case are lower and the wake flow never reaches the supersonic values. The other solution fields such as temperature, normalized pressure, etc. are distictly different from that of the full scale model. The above observations confirms that similar solutions are obtained by matching M_∞, γ, Re_∞ and Pr, thus verifying the findings of dimensional analysis.

4 Wake Flow Shock and Wake Flow Thermal Inversion

The fluid dynamic phenomena associated with a given flow geometry can be viewed as a consequence of the relative strengths of the similarity parameters. The flow dynamic effects of the Beagle II geometry are investigated first by changing the free stream Reynolds, Mach and Prandtl numbers away from the datum point while keeping the remainder of the similarity parameters and total enthalpy the same. The results of these investigations are presented in figure 3 where the results are plotted in the form of the maximum reverse flow Mach number and the base stagnation temperature normalized by total free stream temperature. This format allows us to see the maximum values recovered by the thermal and kinetic components by the wake flow. We call this diagram *wake energy recovery plot* (WERP). Two turbulent models and a reacting flow simulation are also studied. Finally, the total specific enthalpy of the flow is perturbed from the datum value while keeping the same M, γ and free stream density. Figure 3 shows that Re_∞ and M_∞ data lies on one class of curve and Pr data lies in a different class of a curve.

The Prandtl numbers of the simulations are changed from the datum by changing the thermal conductivity k_0 while other similarity parameters are kept constant. The Prandtl number therefore is altered throughout the entire domain. The total temperature of the base flow (thus the base stagnation temperature) approaches the free stream total temperature as the base Prandtl number approach unity. Note that the base Prandtl number is taken to be the Prandtl number in the vicinity of the dividing streamline in the wake region and is not a precise value. For base Prandtl numbers higher than approximately unity, the wake total temperature exceeds that of the free stream. We label this flow condition *wake flow thermal inversion* (WFTI). The Prandtl curve of figure 3 has a negative slope. This is due to the increased temperature of the base flow reducing the maximum base Mach number, M_{bmax} .

We speculate that the Reynolds curve of Figure 3 indicates that the base flow responds to increased availability of mechanical energy of the free stream, due to an increase in Reynolds or Mach free stream values, by receiving more work done on it via viscous forces. This is reflected in the higher value of the maximum base flow Mach number, M_{bmax}, and the temperature ratio of the base centre temperature to the total free stream temperature, T_{base}/T_{tot}, recovered at the centre of the base. As Reynolds or Mach numbers are increased beyond a critical point, M_{bmax} would exceed unity leading to the *Wake Reverse Flow Shock* (WRFS) as the flow transit back to subsonic speeds near the base stagnation point.

The effect of perturbing the free stream total enthalpy is also studied. Figure 3 shows that a 50% increase in total enthalpy would shift the datum point considered to the left and a decrease of 50% would shift it to the right. The effect on T_{base}/T_{tot} is not as

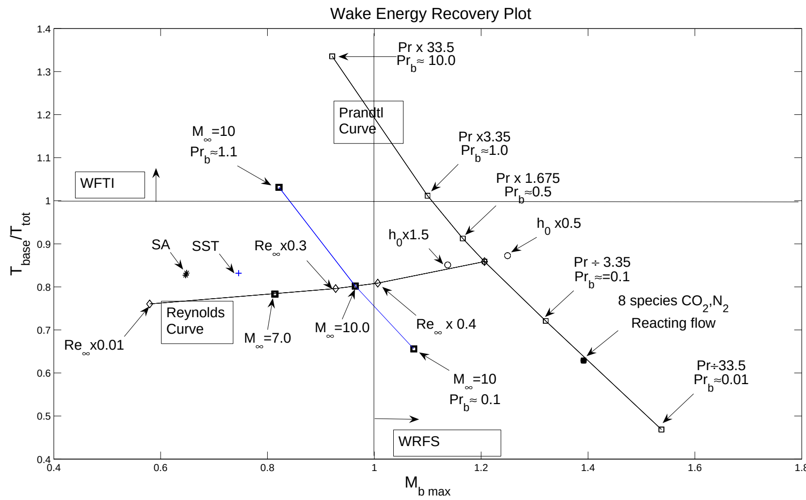

Fig. 3. The wake energy recovery plot (WERP)-The plot of wake flow maximum Mach number vs the ratio of base stagnation temperature to total temperature for changes of Re_∞,M_∞ and Pr. The point of intercept of the Reynolds and Prandtl curves is the datum re-entry condition described in the section 1.

great as on M_{bmax} and the two points lie close to the Reynolds (and Mach) curve. This observation is consistant with the resulting changes in Re_∞ and M_∞ due to the change in the free stream enthalpy.

Based on the above observations we argue that the base flow dynamics associated with the Prandtl number are different from that of the Mach and Reynolds numbers. We propose that while the Mach and Reynolds numbers controls the availability of the kinetic energy of the free stream for the viscous forces to work on the trapped base flow, the Prandtl number governs the balance of the energy in the base flow. At steady-state, in laminar flow, the energy balance of the wake flow is between the work done on it due to viscous interactions and heat transferred out of it due to thermal conductivity. The Prandl number forms the natural non-dimensional parameter for this interaction indicating where the balance lies. This role of the Prandtl number is further supported by the observation that WFTI occurs at base Prandtl number equal to unity.

The effect of turbulence models is investigated for both the Spallart-Almaras model and SST model. In both cases the turbulent viscosity is increased in the base region, mostly near the wake neck region, decreasing the effective Reynolds number of the base region flow. This is also evident through the reduction of the wake length. Both turbulent solutions occupy points on Figure 3 which correspond to lower Reynolds number and higher Prandtl number to the datum point. This agrees with the concept of increased viscosity due to the addition of turbulent viscosity overpowering the increase of effective thermal conductivity due to eddy diffusion. The WRFS does not appear in these turbulent models due to reduced Re. An eight species reaction model for $97\%CO_2$ and $3\%N_2$ by mass fraction [1] [4] has also been computed and occupies a position on figure 3 that correspond to a low Prandtl number. This can be explained by the reduction of the temperature due to the predominance of endothermic reactions in the base flow as well as on the reduction of the effective Prandtl number due to the increase of *reaction conductivity*, a mode of heat transfer due to mass diffusion.

5 On the uses of the Wake Energy Recovery Plot

Representations similar to WERP in figure 3 could be used by space probe design teams in the analysis of error in CFD computations as they describe the interaction of the *flow parameters* (T_{base}/T_{tot}, M_{bmax}), the *similarity parameters* (Re, Pr) and *flow phenomena* (WRFS, WFTI). For example, one could determine from figure 3 that the uncertainty in Prandtl number would lead to a greater effect in base heating than say, Reynolds number. Similarly, the effects of numerical artifacts, such as artificial viscosity, once estimated, could be projected on to the quantities of interest. Mesh convergence could also be observed for multiple quantities of interest simultaneously and an appropriate mesh could be made at critical values of the flow phenomena such as the WRFS to capture the shock.

6 Conclusions and Future Work

The present work establishes the WRFS and WFTI phenomena as a theoretical possibility for steady-state, laminar, compressible flow. Their association with Reynolds number and Prandtl number is established. While the actual wake reverse flow shock and the thermal inversion may not be relevant to most of the re-entry flow, the nature of the Reynolds and Prandtl number effect on the wake flow energy distribution and thus the base heating remains directly relevant. The intriguing possibility of a part of a gas flow attaining temperatures higher than its free stream total temperature is to be investigated further to find out if any real gas would support the requirements. The narrowing down of the thermal and kinetic energy maxima to two classes of curves on WERP significantly constrains the flow dynamics possibilities of a given wake flow. The dependence of the curves on the shape of the re-entry configuration is being investigated. The WERP would have consequences in development of empirical formulae for base heating. This work is supported by ARC DP0666941.

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